Econ 2261 – Problem Set III

Provide a full justification for all your answers.
Due via OWL on 4/3 before 23:50 EST (UTC-5).

1. The government must decide whether to *build* a bridge or *not*. In order to build the bridge, the government must spend 10 million dollars. There are $n > 2$ individuals. Individual i's private benefit from building the bridge is v_i.

(a) When is it efficient to build the bridge?

(b) Find the VCG transfers for this problem.

(c) Is there an efficient mechanism for this problem which *never* runs a deficit?

2. Anna and Bob have two cars, a red car and a blue car. They need to decide who gets the red car, and who gets the blue car. Their values for each of the two cars are summarized in Table 1.

(a) What is the efficient outcome?

(b) Find the VCG transfers for this problem.

(c) Is there an efficient mechanism for this problem which *never* runs a deficit?

	Red	Blue
Anna	v_A(Red)	v_A(Blue)
Bob	v_B(Red)	v_B(Blue)

Table 1 – Preferences over cars

3. Suppose that the drilling rights for a specific location are being auctioned. There are two bidders. Suppose that the value of the oil field is either high ($v = 100$) or low ($v = 0$), with each of these values being equally likely. Moreover, suppose that each bidder observes a signal x_i that could be promising or discouraging. Signals are noisy. Conditional on the value of the field being high, the probability of an optimistic signal is 3/4. Conditional on the value of the field being low, the probability of an optimistic signal is 1/4.

(a) What is the expected value of the field for a bidder who observes a high signal? [*hint*: use Bayes rule]

	a=B	a=C	a=D
$v_B(a)$	v_B	-2	-1
$v_C(a)$	-5	v_C	-5
$v_D(a)$	-1	-2	v_D

Table 2 – Negative consumption externality

(b) What is the expected value of the field for a bidder who observes a high signal, and realizes that the other bidder received a pessimistic signal?

(c) Suppose that the drilling rights are auctioned via a sealed-bid second-price auction, and the two bidders are rational. Would the bidders bid a number greater than, less than, or equal to their expected value for the field?

4. Suppose an object is to be allocated to either Bob, Charlie, or David. Whoever receives the object, it will generate consumption externalities for the other individuals in accordance with Table 2.

(a) What is the efficient outcome?

(b) Find the VCG transfers for this problem.

(c) Is there an efficient mechanism for this problem which *never* runs a deficit?

5. Consider the roommate problem from the lecture notes. For each of the following mechanisms, indicate whether it is incentive compatible for Gary, incentive compatible for Frank, budget balanced and/or Pareto efficient.

(a) First, Gary reports how much he is willing to pay ($\hat{v}_G$). Then, after hearing Bob's report, Frank announces how much he is willing to pay ($\hat{v}_F$). They buy the machine if and only if $\hat{v}_F + \hat{v}_G \geq 1000$. If they but the machine, Frank pays $1000 - \hat{v}_G$, and Gary pays $\hat{v}_G$.

(b) Frank and Gary simultaneously report their values. Each pays 500, regardless of the reports, and regardless of whether they buy the machine. They buy the machine if and only if $\hat{v}_F + \hat{v}_G \geq 1000$.

(c) They buy the machine and Frank pays 1000, if Frank's value is greater or equal than 1000. Otherwise, they do not buy the machine and nobody pays anything.

(d) Frank and Gary simultaneously announce their values. They buy the machine if $\hat{v}_F \cdot \hat{v}_G \geq 250000$. If they buy the machine, Gary pays $250,000/\hat{v}_F$ and Frank pays $250,000/\hat{v}_G$.

(e) Frank and Gary simultaneously announce their values. They buy the machine if $(\hat{v}_F - 800)^2 + (\hat{v}_G - 800)^2 \leq 40000$. If they buy the machine, each pays

$$t_i = 800 - \sqrt{40000 - (y - 800)^2}$$

[*hint*: draw a picture of this mechanism]