Solution Concepts 1
Dominance and best responses
Watson §6, pages 51-64

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Best responses and rationality

- The assumption that motivates our predictions is that players are rational, in a game context this means that: players always choose strategies that maximize their expected utility given their beliefs

Prediction

Given a strategic form game, players will only choose strategies that are a best response to some belief about his/her opponent's strategies

- We use the symbol BR_i to denote the set of such strategies:

$$BR_i = \left\{ s_i \in S_i \;\middle|\; \text{there is some } \theta_{-i} \text{ such that } s_i \in BR_i(\theta_{-i}) \right\}$$

- The prediction is that every player i will choose a strategy in BR_i

Example: A 3×2 game

Best responses

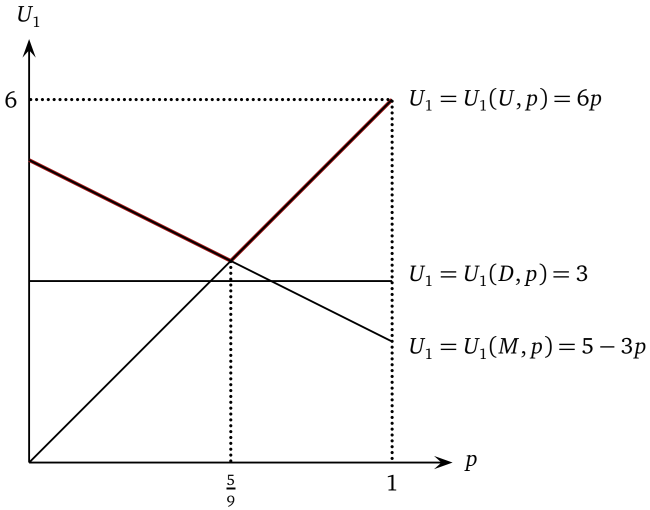
	L p	R $[1-p]$
U	6 , 3	0 , 1
M	2 , 1	5 , 0
D	3 , 2	3 , 1

- When one player has only two strategies, we can graph the expected utility of his/her opponents to find the set of best responses
- Player 1's expected utility is given by:

$$U_1(U,p) = 6p \qquad U_1(M,p) = 5 - 3p \qquad U_1(D,p) = 3$$

Example: A 3×2 game

Best responses



Strictly dominated strategies

motivation

- For general games finding the set of best responses is not that straightforward
- We will find such set indirectly by introducing the notion of *strictly* dominated strategies
- Strictly dominated strategies was originally thought as an interesting concept on its own
- We will use it only because of its relationship with best responses: *a strategy is a best response to some belief if and only if it is not strictly dominated*

Mixed strategies

- Before defining strict dominance we extend our notion of strategy by allowing players to make random choices

Definition

A mixed strategy for player i is a probability distribution σ_i over his/her strategies

- Mathematically, the notions of beliefs and mixed strategies are similar but the interpretation is different
- For example, in a game with two players 1 and 2
  - θ_2 represents 1's beliefs about 2's behavior which might very well be deterministic
  - σ_2 represents 2's behavior which might very well be unknown by 1
- As before, we can compute i's expected utility for playing according to σ_i, $U_i(\sigma_i, s_{-i})$ or $U_i(\sigma_i, \theta_{-i})$

Strictly dominated strategies

Definition

We say that a pure strategy s_i is strictly dominated by a pure or mixed strategy σ_i if playing according σ_i generates a **strictly** higher expected payoff for i than s_i, **independently of what the other players do**. That is, if and only if:

$$U_i(\sigma_i, s_{-i}) > u_i(s_i, s_{-i})$$

for every $s_{-i} \in S_{-i}$.

Example: A 3×2 game

Dominated strategies

	L	R
U	6 , 3	0 , 1
M	2 , 1	5 , 0
D	3 , 2	3 , 1

- For player 2, R is strictly dominated by L because:

$$u_2(U,L) = 3 > 1 = u_2(U,R)$$
$$u_2(M,L) = 1 > 0 = u_2(M,R)$$
$$u_2(D,L) = 2 > 1 = u_2(D,R)$$

Example: A 3×2 game

Dominated strategies

	L	R
U	6 , 3	0 , 1
M	2 , 1	5 , 0
D	3 , 2	3 , 1

- For player 1, D is not strictly dominated U nor by M but it is strictly dominated by $\sigma_1 = (1/3, 2/3, 0)$ because:

$$U_1(\sigma_1, L) = \frac{1}{3}6 + \frac{2}{3}2 = \frac{10}{3} > 3 = u_1(D, L)$$
$$U_1(\sigma_1, R) = \frac{2}{3}5 = \frac{10}{3} > 3 = u_1(D, R)$$

Dominance and best responses

Theorem

*A strategy s_i is a best response for some belief of player i if and only if it is **not** dominated by any other **pure or mixed** strategy*

- Our first prediction was that rational players always choose best responses
- This theorem allows us to determine the set of best responses by *eliminating* the strategies that are strictly dominated
- In many cases (almost surely in the exams) it is sufficient to look for strategies that are dominated *by pure strategies*
- In some few cases, eliminating dominated strategies is sufficient to *fully* predict the outcome of a game

Example: prisoner's dilemma

dominated strategies

	Keep Silent	Confess
Keep silent	$-1, -1$	$-5, 0$
Confess	$0, -5$	$-3, -3$

- In the prisoner's dilemma, keeping silent is strictly dominated by confessing
- We thus can predict that *rational* players playing the prisoner's dilemma will confess