Econ 4020 – Guided Exercise on Perfect Recall

Perfect recall is the requirement that all players have perfect *memory* with regard to both everything they do and everything they observe. If a player knows a piece of information or takes an action at some information set, then he must be able to recall this fact at any further point in the game.

One way to verify whether an information structure satisfies perfect recall is to explicitly write down what each player knows at each point in the game as follows:

1. Draw and label *all* information sets of the game, even those corresponding to a single node.
2. For each information set, write down all the paths (sequences of moves) that could have lead to this information set.
3. Use this list of paths to write down *everything* that the player making choices knows, and everything he does not know.
4. Identify pairs of nodes that can be problematic, i.e., that belong to the same player and that follow one another.
5. If in any of these nodes the corresponding player does not know which action he took in the past, this is a violation of perfect recall.
6. If in any of these pairs of nodes the player knows something in the preceding information set, but not on the subsequent information set, then you have found a violation of perfect recall.
7. If you haven't found any violations of perfect recall in steps 5 and 6, then the game satisfies perfect recall.

Hopefully, you won't have to do this once you get some practice. In fact, you will not have enough time to do anything like this. However, if the concepts of information sets and/or perfect recall appear confusing, it might help you to follow this procedure at least once. My suggestion is that you try to follow these steps *on your own* for the game tree in the following page. Only *after* you are done, compare your work with my solution (starting on page 3 of this document).

If you prefer to work with more formal definitions, see Ch. 11 in *Martin J. Osborne & Ariel Rubinstein* (1994) A Course in Game Theory. It can be downloaded for free from the website of one of the authors at http://books.osborne.economics.utoronto.ca

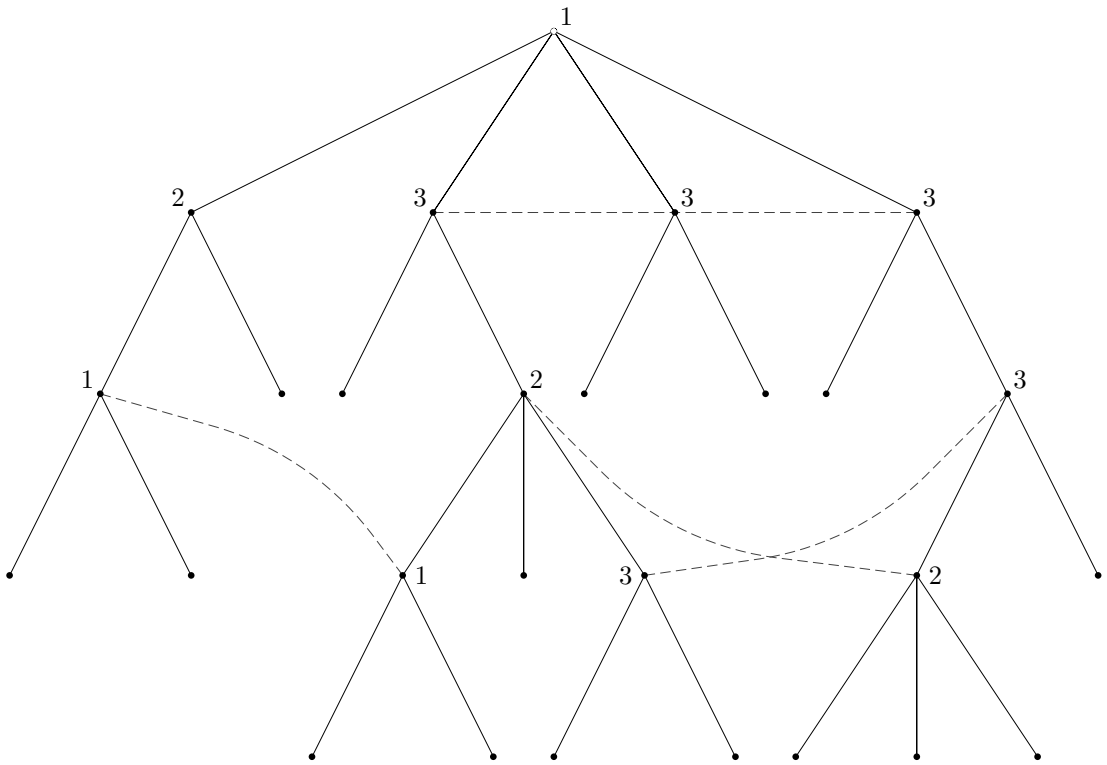

Figure 1 – Is this a valid tree with a valid information structure satisfying perfect recall?

Solution

1. There are 8 information sets, all of which have been labeled in Figure 2.

2. These are the paths leading to each of them

1A This is he beginning of the game

1B Player 1 moving left and then player 2 moving left (1L,2L), or (1ML,3R,2L)

2A (1L)

2B (1ML,3R) or (1R,3R,3L)

3A (1ML) or (1MR) or (1R)

3B (1ML,3R,2R) or (1R,3R)

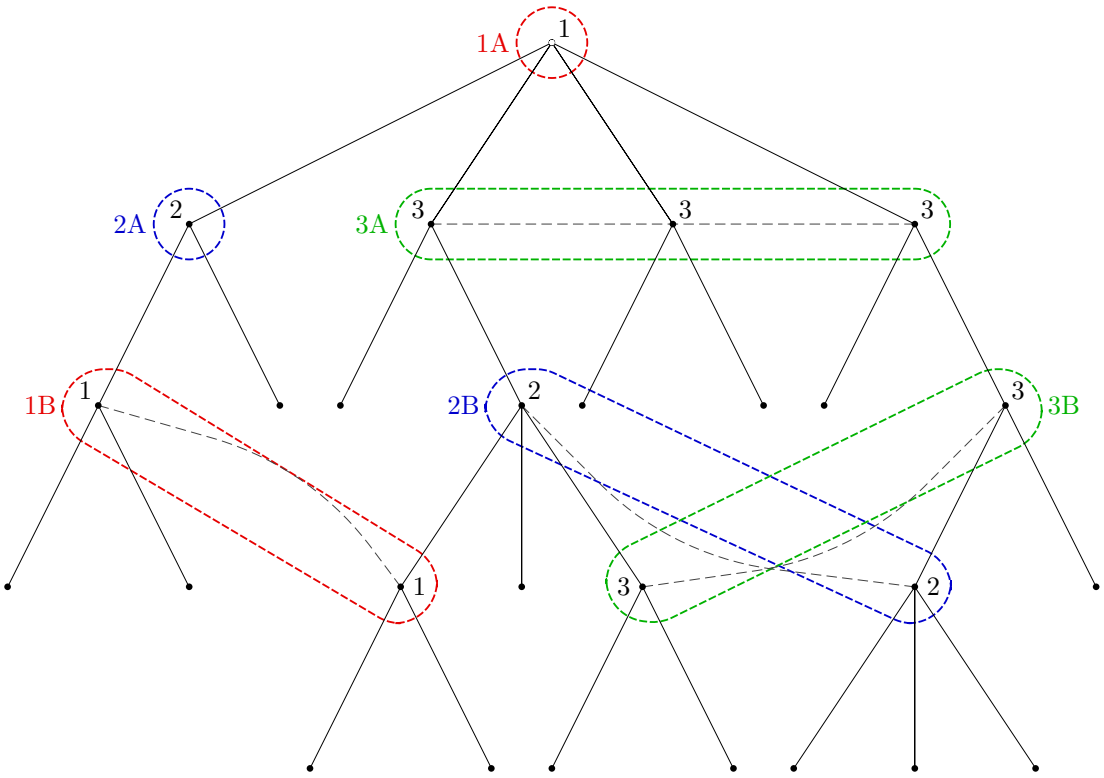

Figure 2 – Information sets

3. Here is what the moving player knows at each information set:

1A Player 1 knows that this is the beginning of the game, and that he has four choices available. Nothing else has happened, so there is nothing about the history of the game that he doesn't know.

1B Player 1 knows that it is his turn to make a decision and he has two choices available. He knows that both player 1 (himself) and player 2 have moved in the past. He knows that player 1 did not choose middle-right nor right at the beginning of the game. He doesn't know whether player 1 chose left or middle-left at the beginning of the game. He knows that player 2 went left. He doesn't know whether player 3 moved or not. He knows that, if player 3 moved, he moved right.

2A Player 2 knows that it is his turn to make a decision and he has 2 choices available. He knows that player 1 moved before him and went left. Nothing else has happened, so there is nothing about the history of the game that he doesn't know (singleton information sets always have this property).

2B Player 2 knows that it is his turn to make a decision and he has 3 choices available. He knows that player 1 moved then player 3 moved. He doesn't know know whether player 3 moved once or twice. He knows that player 1 went either middle-left or right. He knows that player 3 went right and, if he moved twice, went left the second time.

3A Player 3 knows that it is his turn to make a decision and he has 2 choices available. He knows that the only player to move before him was player 1, and that player 1 did not go left. He does not know whether player 1 went middle-left, middle-right, or right.

3B Player 3 knows that it is his turn to make a decision and he has 2 choices available. He knows that player 1 did not go left nor middle-right at the beginning of the game. He does not know whether player 1 went middle-left or right. he knows that player 3 was the second player to move and he went right. He doesn't know whether player 2 has moved. He knows that, if player 2 moved, he went right.

4. The pairs of nodes that could be problematic are (1A,1B) and (3A,3B). In both cases, there is a path going from the former information set to the latter one. The pair (2A,2B) is not problematic because these information sets never take place once after the other. Hence, in this case, there is no possibility for player 2 to forget anything. There are no other pairs that could be problematic, because there are no other pairs of information sets belonging to the same player.

5. From part 3, at 3B, player 3 knows what he did on 3A. However, in information set 1B, player 1 does not know whether he went left or middle-left on 1A. **This is a violation of perfect recall!**

6. Coming into 1A, player 1 didn't know anything that he could forget. From 3, everything that player 3 knows at 3A, he also knows at 3B. In fact, at 3B he knows more about player 1's initial choice than he did at 3A. So, there are no other violations of perfect recall.

7. One violation, like the one we found in step 5, is sufficient to conclude that the information structure does *not* satisfy perfect recall.