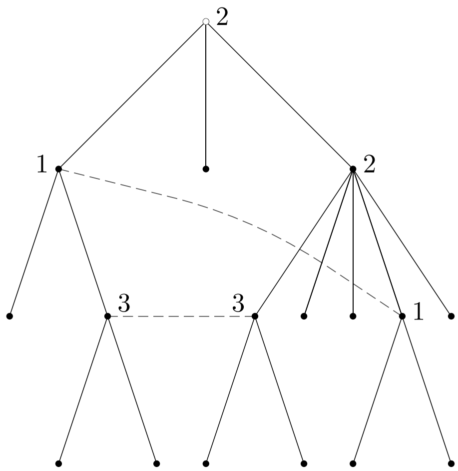
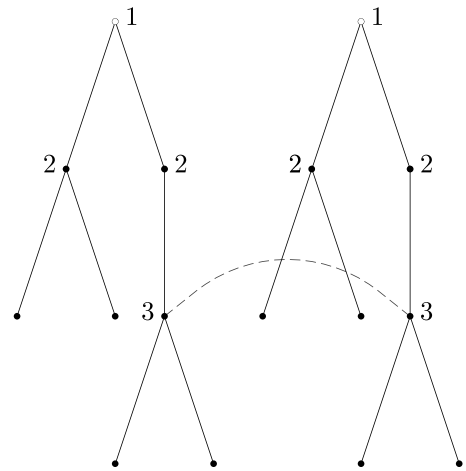
Econ 4020 – Problem Set I – Answer Key

Due on 02/23

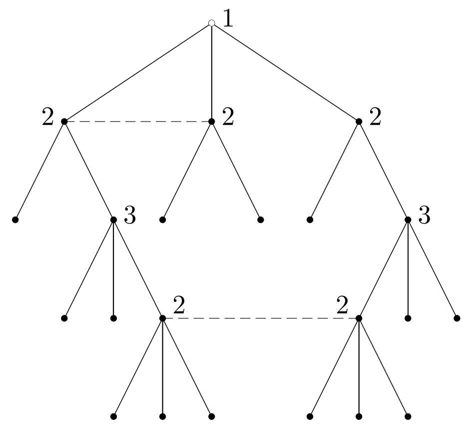
1. Determine which of the following are valid game trees with valid information structures. For the ones that are invalid, explain why.



(a)



(b)



(c)

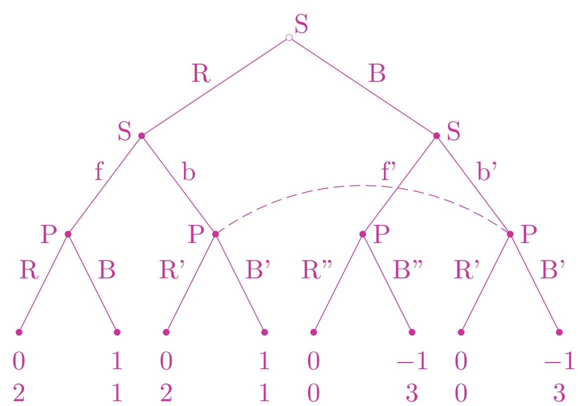
(a) Valid

(b) Not valid because there are two roots

(c) Not valid, because it does not satisfy perfect recall. After player 1 moves, player 2 knows whether player 1 went right or not. Player 2 forget this information later on.

2. Susan and Peter are neighbors and are both getting ready to attend an event. Each of them has to choose between wearing red and wearing blue. Susan prefers to wear a red outfit and Peter prefers to wear blue. Each receives additional utility of 1 from wearing his or her preferred color. In addition, Peter wants to match Susan and he gets an additional utility of 2 from matching. Meanwhile, Susan does *not* want to match Peter and gets a disutility of -1 if they match. The timing is as follows. Susan chooses her outfit first and leaves the house. She can choose to walk out the front door so that Peter can see which outfit she is wearing, or leave through the back door in which case Peter cannot see anything. After this, Peter chooses his outfit.

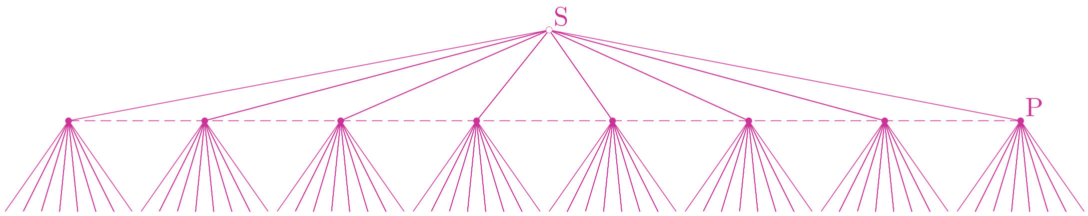
(a) Write down an extensive form game representing this situation.



(b) Write down a strategic form game representing this situation.

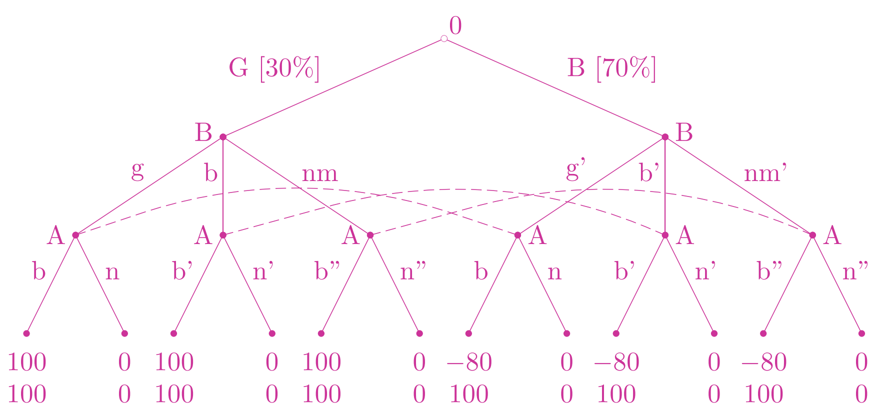
	RR’R”	RR’B”	RB’R”	RB’B”	BR’R”	BR’B”	BB’R”	BB’B”
Rff’	$0, 2$	$0, 2$	$0, 2$	$0, 2$	$1, 1$	$1, 1$	$1, 1$	$1, 1$
Rfb’	$0, 2$	$0, 2$	$0, 2$	$0, 2$	$1, 1$	$1, 1$	$1, 1$	$1, 1$
Rbf’	$0, 2$	$1, 1$	$0, 2$	$1, 1$	$0, 2$	$1, 1$	$0, 2$	$1, 1$
Rbb’	$0, 2$	$1, 1$	$0, 2$	$1, 1$	$0, 2$	$1, 1$	$0, 2$	$1, 1$
Bff’	$0, 0$	$-1, 3$	$0, 0$	$-1, 3$	$0, 0$	$-1, 3$	$0, 0$	$-1, 3$
Bfb’	$0, 0$	$-1, 3$	$0, 0$	$-1, 3$	$0, 0$	$-1, 3$	$0, 0$	$-1, 3$
Bbf’	$0, 0$	$0, 0$	$-1, 3$	$-1, 3$	$0, 0$	$0, 0$	$-1, 3$	$-1, 3$
Bbb’	$0, 0$	$0, 0$	$-1, 3$	$-1, 3$	$0, 0$	$0, 0$	$-1, 3$	$-1, 3$

(c) Write a different extensive form game with the same strategic form. Players simultaneously choosing between 8 strategies can generate the same strategic form game



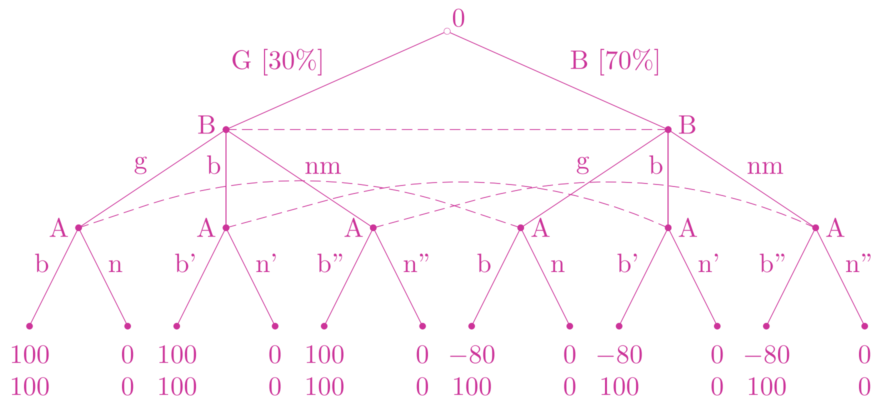
3. Anna is considering buying wine from Bob for \$100. The wine is good with probability 30% and bad with probability 70%. Ana's utility from consuming wine is \$200 if it is good and \$20 if it is bad. After tasting the wine to determine its quality, Bob has the option to send a message to Anna stating that the wine is good, send a message saying that the wine is bad, or not sending a message at all. Talk is cheap: there is no cost for Bob to send a message, and he can lie if he wants to (but he can send at most one message). Anna observes the message but not the quality of the wine before choosing to buy it or not.

(a) Write down an extensive form game representing this situation.



(b) Now suppose that Bob is a terrible sommelier and cannot tell good wine from bad wine. He learns nothing from the tasting, but still can send a message to Anna. Write down an extensive form game representing this situation. See the figure at the top of next page.

(c) How many strategies does each player have in each case? In part (a), Anna has $2^3 = 8$ strategies, while Bob has $3^2 = 9$ strategies. On part (b), Bob only has 3 strategies.



4. David's utility for money is given by $u(x) = x^\alpha$, where $\alpha = 1/2$ is a parameter. David's current wealth is $\omega = \$9$. He is offered to make a bet on the outcome of Cornell's next football game. He will get paid \$7 if he wins the bet, and will have to pay \$9 if he loses the bet. He can either bet that Cornell will win, or bet that Cornell will lose. He believes that Cornell will win with probability $p \in (0, 1)$.

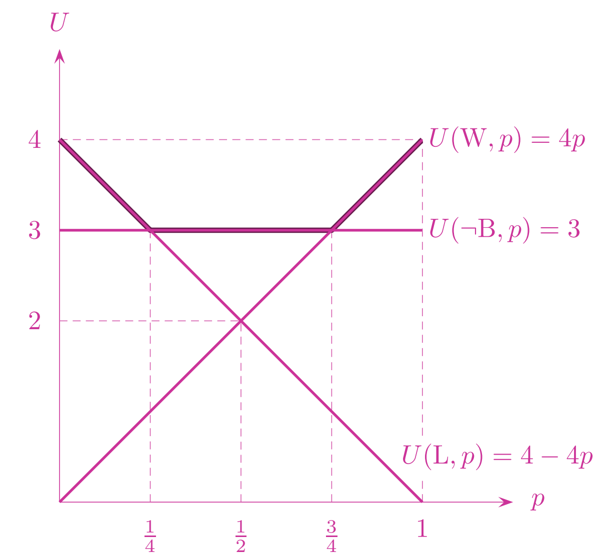
(a) Write down an expression for David's expected utility for betting for Cornell, betting against Cornell, and for not betting at all. Let 'W' represent a bet that Cornell wins, 'L' a be that Cornell loses, and '¬B' no betting.

$$U(\mathrm{W}, p) = p(\omega + 7)^\alpha + (1 - p)(\omega - 9)^\alpha = 4p$$
$$U(\mathrm{L}, p) = 4 - 4p$$
$$U(\neg\mathrm{B}, p) = 3$$

(b) Which actions are David's best responses as a function of p? The graph is at the top of next page.

$$\mathrm{BR}(p) = \begin{cases} \mathrm{W} & \text{if} \quad p > 3/4 \\ \mathrm{W} \text{ or } \neg\mathrm{B} & \text{if} \quad p = 3/4 \\ \neg\mathrm{B} & \text{if} \quad 1/4 < p < 3/4 \\ \mathrm{L} \text{ or } \neg\mathrm{B} & \text{if} \quad p = 1/4 \\ \mathrm{L} & \text{if} \quad p < 1/4 \end{cases}$$

(c) Which actions are rational (i.e., a best response to *some* belief)? All.



(d) What happens to the set of beliefs for which *not* betting is optimal when α increases? It decreases.

Justification: The highest probability of wining for which $\neg$B is optimal is the solution to

$$U(\mathrm{W}, p) = 16^\alpha p = 9^\alpha = U(\neg\mathrm{B}, p)$$

Let $\bar{p}(\alpha)$ denote the solution to this equation as a function of α. Then

$$\bar{p}(\alpha) = \left(\frac{9}{16}\right)^\alpha$$

Since $9/16 < 1$, this function is decreasing in α. A similar argument works for the lower probability of wining for which $\neg$B is optimal.

Intuitive explanation: When α increases, the utility function $u(x) = x^\alpha$ becomes 'less concave' (use a computer program to draw a graph to convince yourself). That is, the agent becomes less risk averse. Hence, the region where the safe action ($\neg$B) is optimal decreases.

(e) What happens to the set of beliefs for which *not* betting is optimal when ω increases? (suppose David still loses \$9 if he loses the bet) It decreases.

Intuitive explanation: Let z denote David's earnings ($z = +7$ if we wins the bet, $z = -9$ if he loses the bet, and $z = 0$ if he doesn't bet). We can write David's utility in terms of z as $\hat{u}(z) = (\omega + z)^\alpha$. When ω increases, the utility function $\hat{u}$ becomes 'less concave' in the relevant range. Higher wealth comes with increased tolerance to the same "amount" of risk.

5. Consider the following game in strategic form

	a	b	c	~~d~~
w	0 , 7	2 , 5	7 , 0	~~0 , 1~~
x	5 , 2	3 , 3	5 , 2	~~0 , 1~~
y	7 , 0	2 , 5	0 , 7	~~0 , 1~~
~~z~~	~~0 , 0~~	~~0 , −2~~	~~0 , 0~~	~~10 , −1~~

(a) Which strategies are strictly dominated for each player? Only d is strictly dominated.

Justification: w is a best response to c, x is a best response to b, y is a best response to a, z is a best response to d, a is a best response to w, b is a best response to x, and c is a best response to y. Hence, none of these strategies is strictly dominated. Strategy d is dominated by the mixed strategy that mixes a and c with equal probabilities.

(b) Is there any belief for which strategy d is a best response? No, since it is strictly dominated.

(c) Is there any belief for which strategy z is a best response? Yes, for instance, the beliefs that assigns full probability to d.

(d) Find all rationalizable strategies for each player. Once d is eliminated, z becomes dominated by x. After that, there are no more dominated strategies. Hence, the rationalizble strategies are a, b, and c for the column player, and w, x and y for the row player.

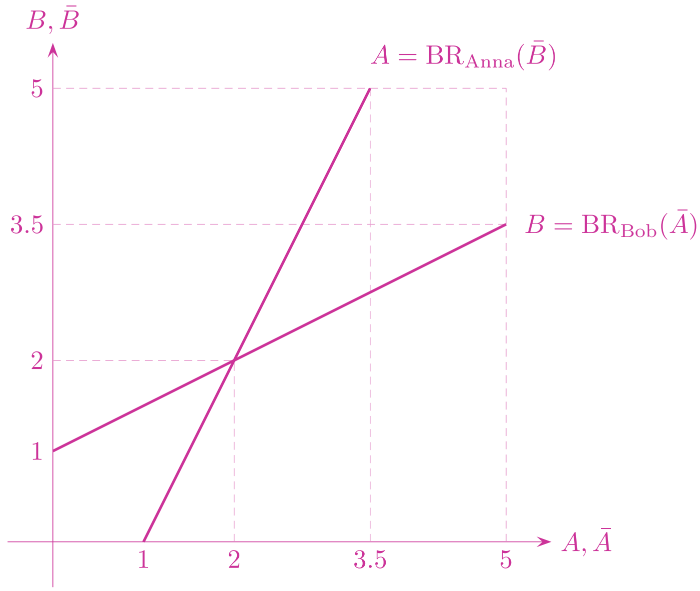
6. Anna and Bob work as partners. The firm's revenue depends on the level of effort provided by each of them. Each of them can provide any level of effort in $[0, 5]$. Let A denote the level of effort provided by Anna, and B the level of effort provided by Bob. Providing effort is costly. The cost for Anna is $-A^2$ and the cost for Bob is $-B^2$. The total revenue of the firm equals $4A + 4B + 2AB$. Anna and Bob receive half the firm's revenue each.

(a) Write down a strategic form game representing this situation. Who are the players? What strategies do they have available? What are the payoff functions?

- The players are Anna and Bob

- Anna chooses $A \in [0,5]$ and Bob chooses $B \in [0,5]$
- $u_{\text{Anna}}(A,B) = 2A + AB + 2B - A^2$ and $u_{\text{Bob}}(A,B) = 2A + AB + 2B - B^2$

(b) Find an analytic solution for the best response functions and graph them in a clearly labeled figure. The first order condition for Anna is $2 + B = 2A$, which implies $\text{BR}_{\text{Anna}}(B) = 1 + B/2$. Similarly, the best response function for Bob is $\text{BR}_{\text{Bob}}(A) = 1 + A/2$.



(c) Can Anna rationalize choosing $A = 4$? How about $A = 2.5$ or $A = 1.5$? Justify your answer *in detail*. No, none of those effort levels are rationalizable. *Justification*: Iterated removal of strictly dominated strategies:

- BR_{Anna} only takes values between $\text{BR}_{\text{Anna}}(0) = 1$ and $\text{BR}_{\text{Anna}}(5) = 3.5$. Hence, $A = 4$ is strictly dominated.
- Given $1 \le A \le 3.5$, BR_{Bob} only takes values between $\text{BR}_{\text{Bob}}(1) = 1.5$ and $\text{BR}_{\text{Bob}}(3.5) = 2.75$.
- Given $1.5 \le B \le 2.75$, BR_{Anna} only takes values between $\text{BR}_{\text{Anna}}(1.5) = 1.75$ and $\text{BR}_{\text{Bob}}(2.75) = 2.375$. Hence, $A = 1.5$ and $A = 2.5$ are *not* rationalizable.

Note that this argument only works because the best response functions are monotone (why?).

(d) Find a level of effort that is rationalizable for Anna. $A = 2$ is rationalizable. *Justification*: $\text{BR}_{\text{Anna}}(2) = \text{BR}_{\text{Bob}}(2) = 2$.

Ü///