Econ 4020 – Problem Set III

Due on 05/09

1. Suppose Anna and Bob play an infinitely repeated Prisoner's dilemma. Their discount factor is $\delta \in (0,1)$. The stage game payoffs are as follows.

	C	D
C	$4,4$	$0,6$
D	$6,0$	$1,1$

In any given history of the game, we say that a player, say Anna, is in good standing if:

- Is the first period of the game
- On the previous period she played C and Bob was in good standing
- On the previous period she played C and she was in bad standing
- On the previous period she played D, she was in good standing and Bob was in bad standing

Consider the modified tit-for-tat strategy profile given by

> *"Play C unless you are in good standing and your opponent is in bad standing, in which case you should play D"*

For which values of δ does this strategy profile constitute a SPNE?

2. Suppose that Anna wishes to hire Bob and Charlie to work in her firm. She can offer each of them a contract consisting of a base wage and a performance bonus. Denote these contracts by (ω_B, b_B) for Bob, and (ω_C, b_C) for Charlie. Reservation wages are $\omega_A^0 = \omega_B^0 = \omega_C^0 = 1$. If both Bob and Charlie accept the contract, they simultaneously choose effort levels $e_B \geq 0$ and $e_C \geq 0$, respectively. In that case, the game ends with payoffs

$$u_A = (1 - b_B - b_C)\pi - \omega_B - \omega_C$$

$$u_B = \omega_B + b_B\pi - e_B^2$$

$$u_C = \omega_C + b_C\pi - 2e_C^2$$

where π is the total firm revenue given by

$$\pi = e_C + e_B + e_C e_B.$$

(a) Find the optimal contract

(b) [Bonus] Is the outcome efficient?

3. Consider a first-price sealed-bid auction. Suppose there are $n \geq 2$ bidders with independent private values distributed uniformly on $[0, 1]$.

(a) Suppose that player 1 bids b_1, and all other players use a bidding function of the form $\beta(v_i) = \alpha \cdot v_i$, where $\alpha \in (0, 1)$ is a constant. What is the probability that player 1 wins the object?
[*Hint*: $\Pr(\text{win}) = \Pr(b_1 \geq \alpha v_i \text{ for all } i \neq 1) = \Pr(\alpha v_2 \leq v_1)^{n-1}$]

(b) Find the unique BNE.

4. Suppose that there are 4 men and 4 women with references given in the following tables

man	preferences
1	$b \succ a \succ c \succ d$
2	$a \succ c \succ b \succ d$
3	$c \succ b \succ a \succ d$
4	$b \succ a \succ c \succ d$

woman	preferences
a	3≻4≻1≻2
b	2≻3≻4≻1
c	4≻1≻2≻3
d	1≻2≻3≻4

(a) Find a matching that is *not* stable.

(b) Use the Gale-Shapley algorithm to find a stable matching.