Extensive Form Games

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Reading assignments: Watson, Ch. 2 & 14

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extensive form games

- Strategic environment
  - Agents make choices
  - Each agent's preferences may depend on the choices of others

- A extensive form game explicitly specifies
  1. Agents involved
  2. Different choices each agent might face
  3. Information available to each agent at each moment
  4. Sequential/temporal structure of choices
  5. Preferences over possible outcomes

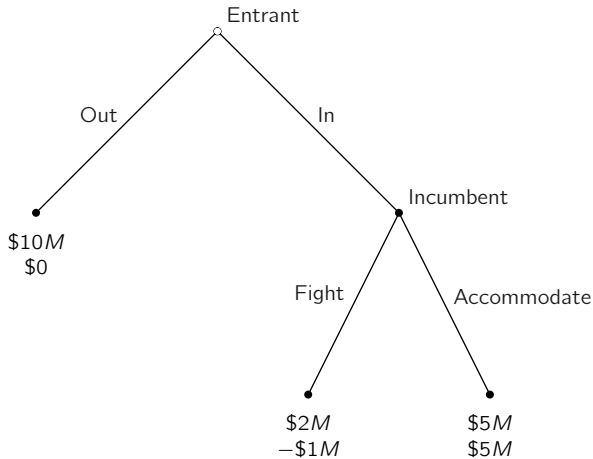
example – entrance deterrence

- Market with a single *incumbent* firm
- Potential *entrant* considers entering

- If the entrant stays out, the incumbent makes $\$10M$ in profits
- If the entrant enters, then the incumbent chooses between
  - *Fighting* the entrant with aggressive policies
  - *Accommodating* the entrant and sharing the monopolistic profits

- If the incumbent accommodates, each makes profits of $\$5M$
- If the incumbent fights, the entrant suffers losses of $-\$1M$ but the incumbent's profits shrink to $\$2M$

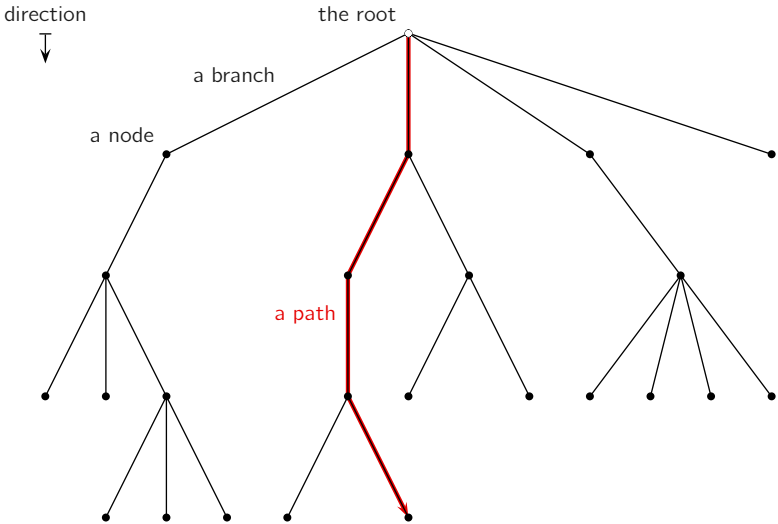
example – entrance deterrence



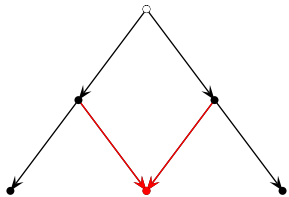
A tree is a set of nodes connected by branches such that

1. A unique node —the *root*— has no incoming branches
2. Every other node has a *unique* incoming branch
3. Every node can be reached starting from the root

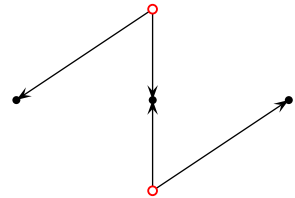
example



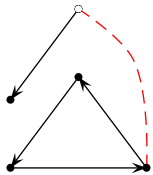
things that are not trees



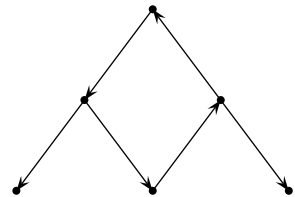
two predecessors



two roots



disconnected



no root

terminal and decision nodes

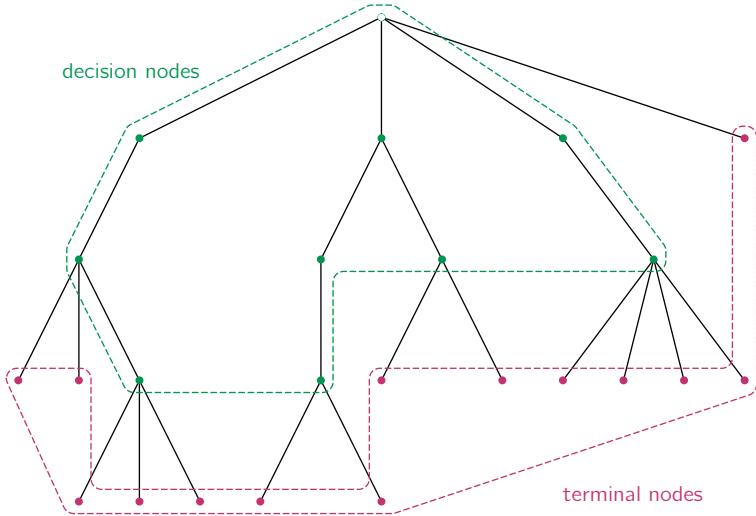
- Trees model dynamic structures

- Nodes represent moments or states of the game
- Branches represent transitions between states via moves or choices

- Two types of nodes

Terminal	Decision
No outgoing branches	At least one outgoing branch
Final states of the game	Initial and intermediate states
No more choices to be made	Some agent is to make a move/choice

example



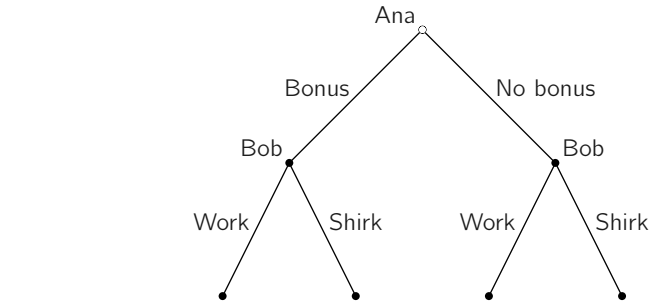
A perfect information extensive form game consists of:

1. A set of players
2. A game tree representing the dynamic structure
3. A specification of who moves at each decision node
4. A payoff for each player at each terminal node

example – performance bonuses

- *Anna* owns a firm that employs *Bob*
- Bob chooses to *work* diligently or *shirk*
- Ana's profits depend on Bob's effort
  - If Bob works, the firm does well and Anna makes $500
  - If Bob shirks, the firm does poorly and Anna only makes $200
- Bob cares about his salary and his effort
  - Working requires costly effort worth $100
  - He receives a fixed salary of $100 independently of his effort
- Before production takes place, Anna has the option of promise Bob a $150 productivity bonus contingent on good results

example – performance bonuses



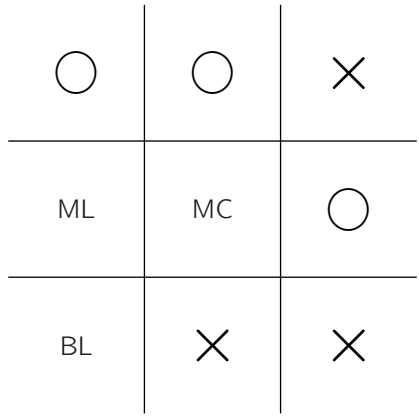
+Profits	+\$500	+\$200	+\$500	+\$200
–Wage	–\$100	–\$100	–\$100	–\$100
–Bonus	–\$150	–	–	–
Ana's payoff	\$ 250	\$100	\$400	\$100

+Wage	+\$100	+\$100	+\$100	+\$100
–Effort	–\$100	–	–\$100	–
+Bonus	+\$150	–	–	–
Bob's payoff	\$150	\$100	\$0	\$100

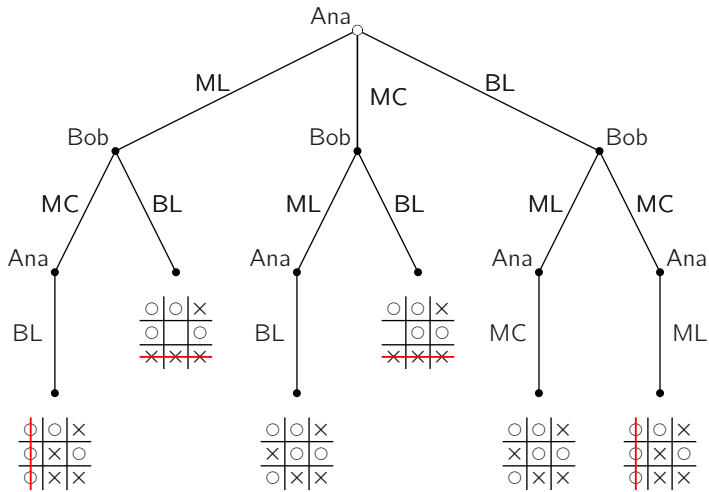
tic-tac-toe

- Tic-tac-toe is a board game played on a 3 by 3 grid
- Two players, Ana and Bob, alternate taking turns
- The player taking a turn marks one free space with his/her mark
- A player wins the game if he/she gets three marks lined up
- If the board is full and there is no winner, the game ends a draw
- The winner's payoff is $+1$, the loser's payoff is -1, and both players get 0 in case of a draw

tic-tac-toe



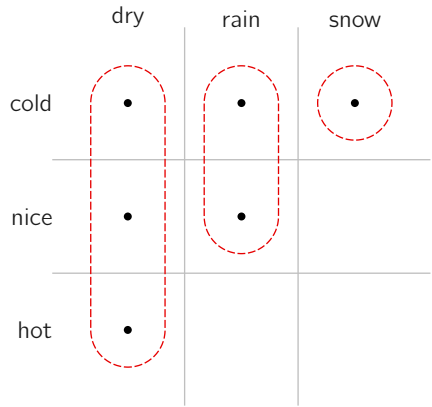
tic-tac-toe


Ana
ML
MC
BL
Bob
Bob
Bob
MC
BL
ML
BL
ML
MC
Ana
Ana
Ana
Ana
BL
BL
MC
ML

- Until now, players always know everything that has happened in the past
- Games with this property are called perfect information games

- In many situations players choose without knowing the state of the game
- Some examples:
  - Make a bet without knowing your opponent's hand
  - Choose which products to develop without knowing your competitor's plans
  - Choose a price without knowing your demand curve

weather through a window

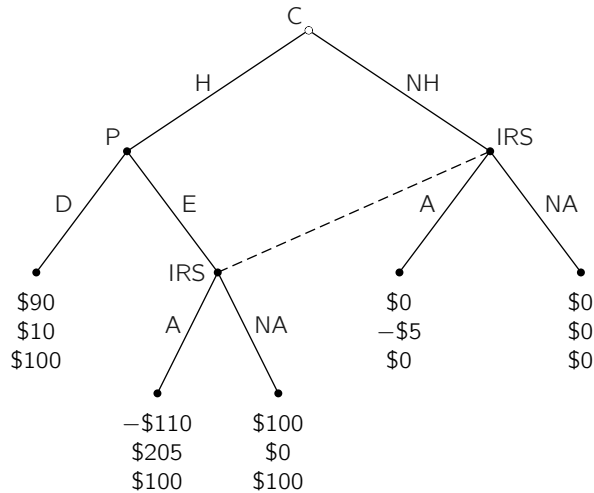


- Through the window you can see precipitations, but not temperature
- For instance, if it is raining
  - You can tell it is raining
  - You cannot tell whether it is nice or cold

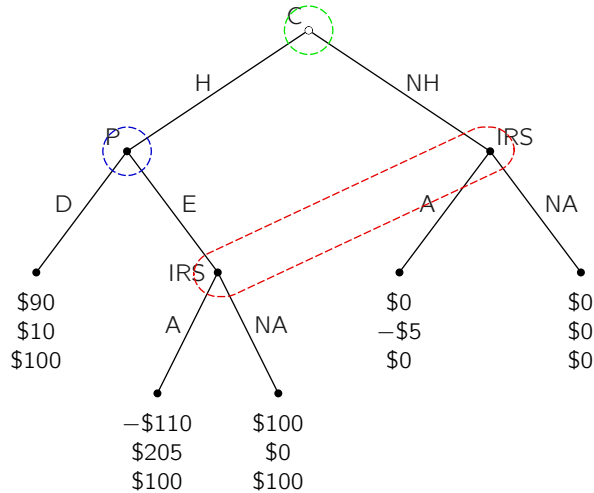
example – collecting taxes

- Paul is a plumber and Charlie is one of his clients
- On a given year she will either hire his services or not
- Her benefit from the service is $200 and she pays him $100
- If hired, Paul chooses whether to declare the sale and pay the IRS $10 worth or taxes
- If the IRS receives no declaration they have the option of either audit Paul or not
- In that case, the IRS does not know whether Paul was not hired or is trying to evade taxes
- Auditing costs $5
- If Paul is caught evading, he pays the IRS the owed taxes plus a $200 fine

example – collecting taxes



example – collecting taxes



information sets

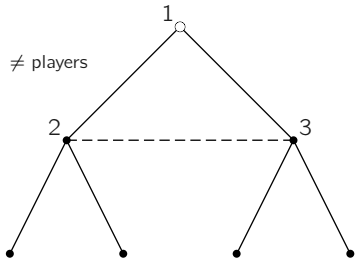
- Information sets describe what player know when making decisions
- Decision nodes that are indistinguishable are grouped together
- Each group is called an information set
- The decision maker knows that the game is in some node within the information set but he/she cannot tell which
- He she must make the same choice in the same way in all the nodes within the same information set

valid information structures

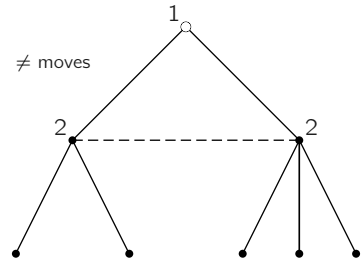
1. Players know when its their turn to make a choice
   - The same player has to move at all the nodes within the same information set
2. Players know which moves they have available
   - All the nodes within the same information set must have the same number of outgoing branches
3. Players never forget any information
   - Both about moves made by others and about their own moves
   - This condition is called perfect recall

invalid information structures

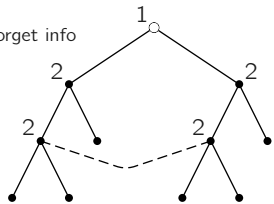
≠ players



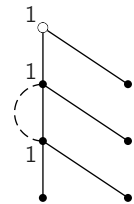
≠ moves



Forget info



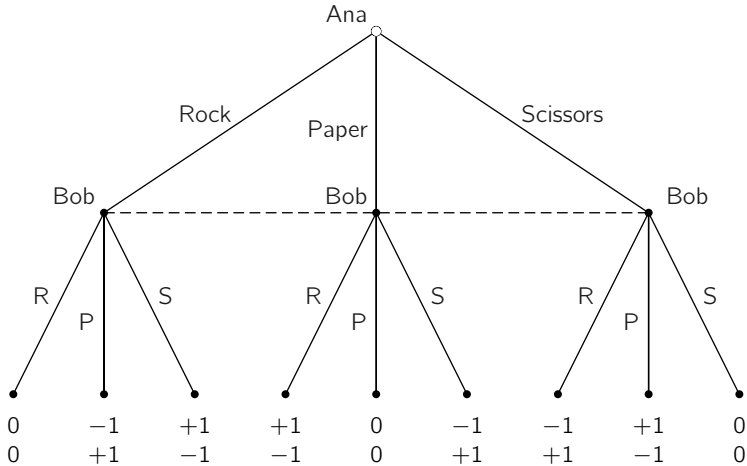
Forget move



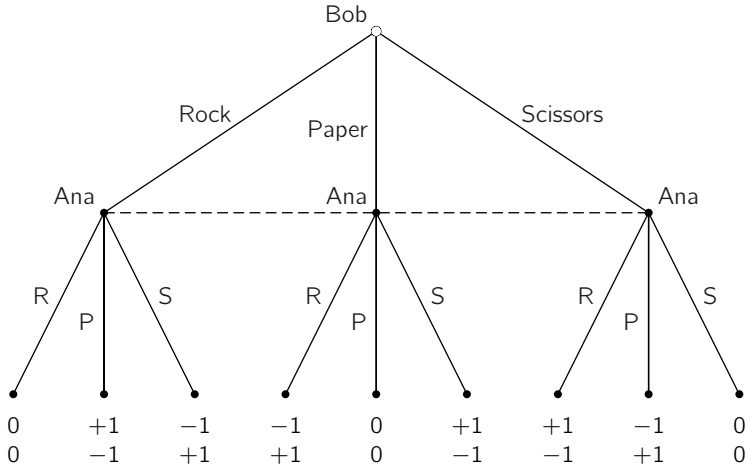
A simultaneous move game is an extensive form game in which

1. Each player makes a single choice
2. Each player has no information about his opponent's choices at the moment of making his own

rock, paper, scissors



rock, paper, scissors



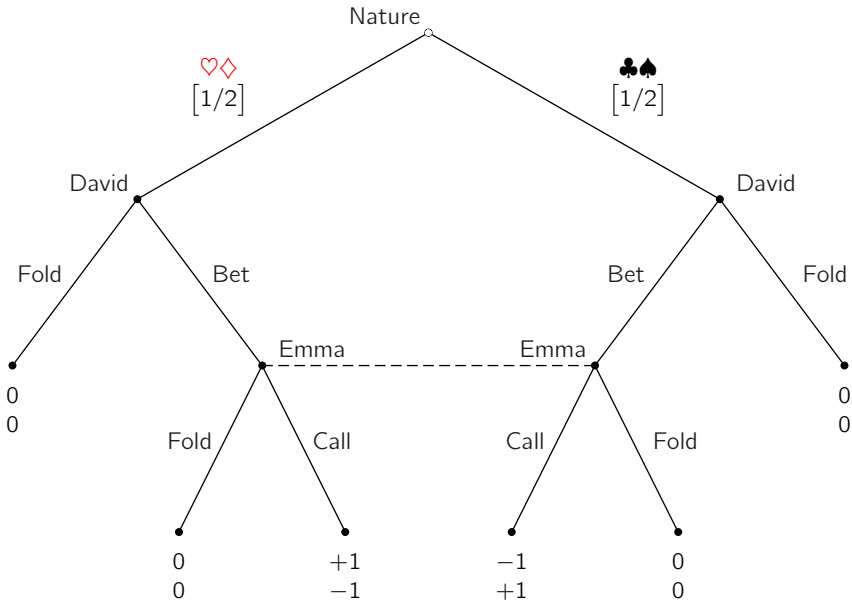
chance

- Some outcomes might be beyond the control of the players, e.g., weather
- Imperfect information about them can be a crucial part of the game
- We model this by adding a non-strategic player called nature or chance
- Nature has actions, but no payoffs
- Instead, we directly specify the probability that it makes each possible action

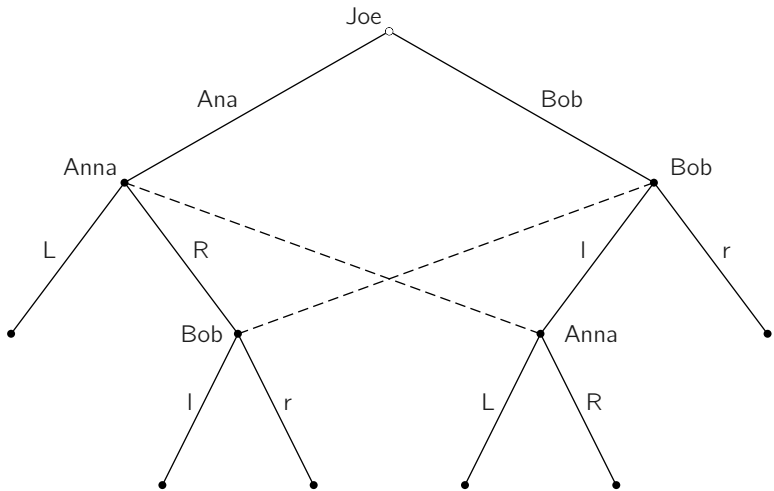
simplified poker

- Nature deals a single card to David
  - A black card with probability 1/2
  - A red card with probability 1/2
- After seeing his card, David decides whether to bet a dollar that it is red
- Seeing the bet but not the card, Emma chooses between calling or folding
- David wins the bet if the card is red, and Emma wins otherwise

simplified poker



An extensive form game consists of:

1. A set of players
2. A game tree representing the dynamic structure
3. A specification of who(either a player or chance) moves at each decision node
4. A valid information structure satisfying perfect recall
5. Probability assignments for chance's moves
6. A payoff for each player at each terminal node

example – a non-timeable tree