Econ 2261 – Homerowk I

Provide a full justification for all your answers.

Due on 02/03

1. Which of the three minimal requirements discussed in class is violated by Borda's rule? Justify your answer thoroughly by providing an explicit instance in which the condition is violated and explaining why it is violated. *Fails IIA.* To see this, we need an example. Suppose there are three alternatives A, B and C, and two individuals. Suppose that the individuals have the following preferences $C \succ_1 A \succ_1 B$, and $B \succ_2 C \succ_2 A$. Following Borda rule, A gets 3 points, B gets 4 points, and thus $B \succ^* A$. Suppose instead that $A \succ_1 B \succ_1 C$, $C \succ_2 B \succ_2 A$. Now, $B \sim^* A$.

2. Consider the following SWF. Fix a preference ordering over alternatives detoned $\succ^0$. For each pair of alternatives A and B, alternative A is considered to be socially preferred to alternative B if and only if $A \succ^0 B$. In other words, $\succ^*=\succ^0$, regardless of the individual's preferences.

(a) Does it satisfy universal domain? *Yes.* $\succ^0$ is fixed and is always well defined.

(b) Does it satisfy unanimity? *No.* If $A \succ^0$ then $A \succ^* B$, even if every individual prefer B to A.

(c) Does it satisfy independence of irrelevant alternatives? *Yes.* The social ranking of every pair of alternatives is fixed and never changes.

(d) Is it a dictatorship? *No.* The ranking $\succ^0$ is fixed and never changes (it is not the preference ranking of an individual).

3. Suppose a group of five friends is going to choose a movie to watch. There are five movies called A, B, C, D, and E, respectively. The friends' preferences are summarized in Table 1.

(a) Which alternatives are Pareto dominated? *C and D.* A dominates D and B dominates C.

(b) Which alternatives are Pareto efficient? *A, B, and E*, that is, those that are not dominated.

(c) What is the social ranking resulting from the Condorcet criterion? $E \succ^* A \succ^* B \succ^* D \succ^* C$.

(d) Can you tell from the information on the table which is the utilitarian alternative? *No.* The utilitarian criterion relies on utility functions. Knowing preferences is not enough.

1	2	3	4	5
E	E	E	E	B
A	A	A	B	A
B	D	B	C	D
C	B	D	A	C
D	C	C	D	E

Table 1 – Preferences over movies

4. Determine which of the following statements are true and which are false.

(a) There always exist a Pareto efficient alternative. *True.* If there are finitely many alternatives, the alternative chosen by Borda rule is Pareto efficient (with infinitely many alternatives it is more complicated).

(b) There always exist a Pareto dominated alternative. *False.* Consider problem 7 (there are no feasible Pareto dominated alternatives).

(c) The best alternative according to Rawlsian Justice is always Pareto efficient. *True.* Suppose that an alternative A is not Pareto efficient. Then there must exist another alternative, say B, that Pareto dominates A. That means that $u_i(B) > u_i(A)$ for *every* individual i. That means that A is *not* the best alternative according to Rawlsian Justice.

(d) The best alternative according to Borda rule is always Pareto efficient. *True.* Suppose that an alternative A is not Pareto efficient. Then there must exist another alternative, say B, that Pareto dominates A. Following Borda rule, B would get more points than A from every individual in society. That means that A is *not* the best alternative according to Borda rule.

5. Consider a competitive industry where all firms have identical cost functions $C(y) = y^2 + 1$ if $y > 0$ and $C(0) = 0$. Suppose that the demand curve for this industry is given by $D(p) = 52 - p$.

(a) Find the individual supply curve of each firm. $MC(y) = 2y$. $AVC(y) = (y^2+1)/y$. $MC(y) \geq AVC(y)$ if and only if $y \geq 1$. Therefore

$$S_j(p) = \begin{cases} p/2 & \text{if } p \geq 2 \\ 0 & \text{otherwise} \end{cases}.$$

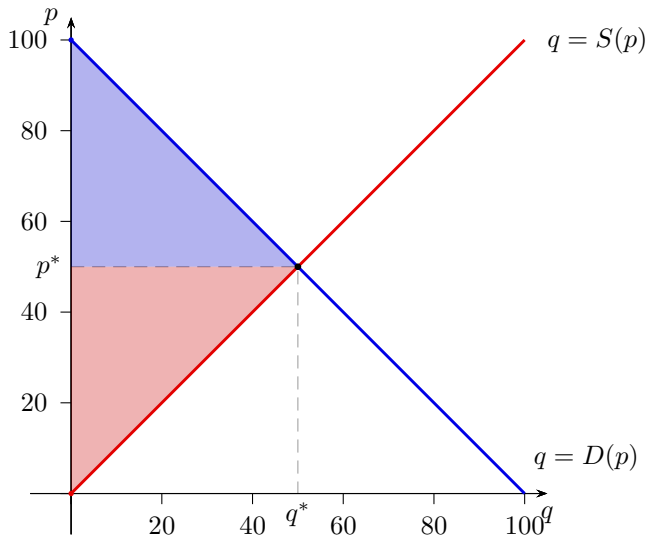

Figure 1 – No taxes

(b) Suppose there are n firms in the industry. Find the industry supply curve. $S(p) = nS_j(y) = np/2$ if $p \geq 0$, and $S(p) = 0$ otherwise.

(c) Find the price and quantity corresponding to a short-run market equilibrium assuming there are n firms in the market. Supply equals demand requires $52 - p = np/2$. This implies $p^* = 104/(2+n)$, and thus $q^* = S(p^*) = 52n/(2+n)$.

(d) Use the zero-profit condition to find the price in the long-run equilibrium. The zero profit condition is $S_j(p)^2 + 1 = pS_j(p)$, or $p^2/4 + 1 = p^2/2$. The only positive solution is $p^L = 2$.

(e) Find the number of firms and equilibrium quantity in the long-run. The long run supply is $S(p^L) = n$. The long run demand is $D(p^L) = 52 - 2 = 50$. Hence, in the long run there are 50 firms, and the total market supply is 50. Each individual firm supplies 1 unit to the market.

6. Let the industry demand be $D(p) = 100 - p$, and the industry supply be $S(p) = p$.

(a) Find the equilibirum quantity and the equilibrium price Setting supply equal to demand yields $p = 50$. Thus $q = 50$.

(b) Draw the demand and supply on a graph. Show on this graph the equilibrium, the consumer surplus and the producer surplus. See Figure 1

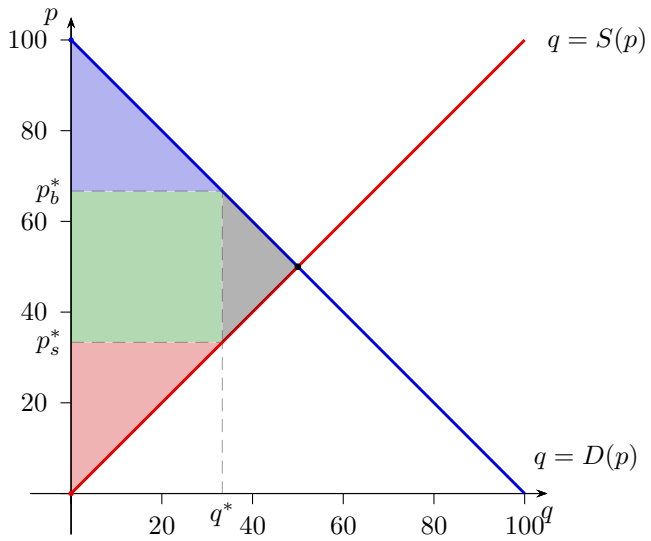

Figure 2 – With taxes

(c) Find the value of the producer surplus. Producer surplus is half area of the rectangle from the origin to the equilibrium. That is, $p^*q^*/2 = 50^2/2$.

(d) Find the value of the consumer surplus. Consumer surplus is also equal to $50^2/2$. See Figure 1.

Now let the government introduce a value tax of 50% paid by the producers. The price the seller faces after the tax is $p_s = p_b - T = p_b - 50\%p_b = p_b/2$. The market clearing condition is thus $D(p_b) = S(p_b/2)$, or $100 - p_b = p_b/2$. This yields $p_b = 200/3$, $p_s = 100/3$, and $q^* = 100/3$. See Figure 2

(e) Find the new consumer's surplus. $(100/3)^2/2$

(f) Find the new producer's surplus. $(100/3)^2/2$

(g) Find the government revenue. $(100/3)^2$

(h) Find the deadweight loss. $(100/3)(50 - 100/3)/2 = 50^2/9$

7. Consider an exchange Economy with a single good and two consumers (Anna and Bob). Suppose that Anna's utility function is $u_A = x_A$ while Bob's utility function is $u_B = 2x_B$. The total endowment of the good consists of one unit.

(a) Find all Pareto efficient allocations. All feasible allocations are Pareto efficient. Consider any allocation. To make Anna better off, one would have to increase her consumption. That requires taking consumption away from Bob, which would make Bob worse off.

(b) Find the utilitarian allocation. The utilitarian allocation is $x_B = 1$, $x_A = 0$. You find this allocation by solving the mathematical problem:

$$\max_{x_A, x_B \geq 0} \quad u_A(x_A) + u_B(x_B) \qquad \text{s.t. } x_A + x_B = 1$$

(c) Find the Rawlsian allocation. The Rawlsian allocation is $x_B = 1/3$, $x_A = 2/3$. You find this allocation by solving the mathematical problem:

$$\max_{x_A, x_B \geq 0} \quad \min\left\{u_A(x_A), u_B(x_B)\right\} \qquad \text{s.t. } x_A + x_B = 1$$

(d) Are the preferences of Anna and Bob any different? Does it make sense to treat them differently? Briefly justify your answer. They have the same preferences (choices).

8. Consider an exchange economy with two goods (1 and 2) and two consumers (Anna and Bob). Anna's utility is $u_A = x_{A1}x_{A2}$, while Bob's utility is given by $u_B = x_{B1} + x_{B2}$. Initial endowments are $(3, 0)$ for Anna, and $(0, 2)$ for Bob.

(a) Find all Pareto efficient allocations. The equitangency condition implies $x_{A1}/x_{A2} = 1$. Hence all the allocations in the line segment from $(0, 0)$ to $(2, 2)$ are Pareto efficient. Also the allocations in the line segment from $(2, 2)$ to $(3, 2)$ are Pareto efficient. The entire set corresponds to the thick red line in Figure 3.

(b) Find the competitive equilibrium. Normalize $p_2 = 1$ and let $p = p_1/p_2$. The first oder conditions from the consumers' optimization problem are $x_{A1}/x_{A2} = p = 1$. Hence $p^* = 1$ and $x^*_{A1} = x^*_{A2}$. Anna's budget constraint thus becomes $x_{A1} + x_{A2} = 2x_{A_1} = 3$. And thus $x^*_{A1} = x^*_{A2} = 1.5$. Feasibility (market clearing) implies $x^*_{B1} = 3 - 1.5 = 1.5$ and $x^*_{B2} = 2 - 1.5 = 0.5$.

(c) Draw a clearly labeled Edgeworth box. See Figure 3.

(d) Are the preferences of Anna and Bob any different? Does it make sense to treat them differently? Briefly justify your answer. Yes. They have different preferences, and thus it makes sense to treat them differently.

[*Hint*: don't forget to check the boundaries of the Edgeworth Box]

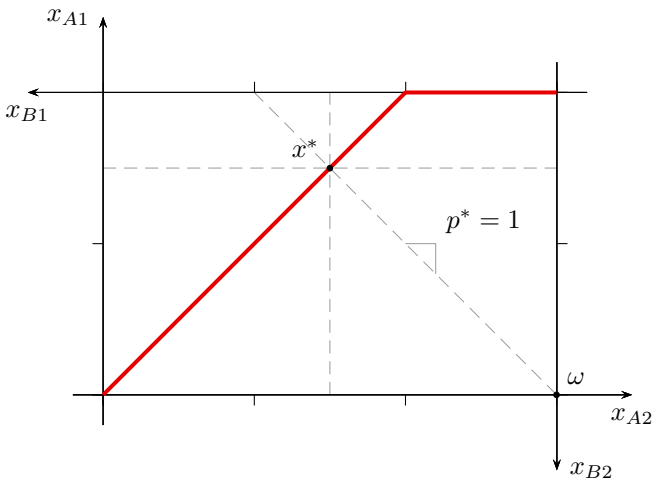

Figure 3 – Edgeworth Box