Econ 2261 – Problem Set II

Provide a full justification for all your answers.
Due on 03/09

1. Consider a monopolist with the cost function $C(q) = 6q$, facing the market demand function $D(p) = 20 - 2p$.

(a) Find the monopoly quantity and price, the monopolist's profit and the consumer surplus. Inverse demand is $P(q) = 10 - q/2$. Profit function is $\pi(q) = (10 - q/2)q - 6q = 4q - q^2/2$. First order condition yields $q = 4$. Price is $p = P(4) = 8$. Profits are $\pi(4) = 8$. Consumer surplus is $\text{CS} = (10 - p)q/2 = 4$.

(b) Now suppose that the government gives to the monopolist a subsidy of \$2 per unit sold. Find the monopoly quantity and price, the monopolist's profit, the consumer surplus, and the cost of the subsidy. With the subsidy $p_s = p_b + 2$. The profit function is thus $\pi(q) = (10 - q/2 + 2)q - 6q = 6q - q^2/2$. First order condition yields $q = 6$. Price is $p = P(6) = 7$. Profits are $\pi(6) = 18$. Consumer surplus is $\text{CS} = (10 - p)q/2 = 9$. The cost of the subsidy is $k = 2q = 12$.

(c) How does this subsidy affect total surplus (taking into account its cost)? The total surplus without the subsidy is $\pi + \text{CS} = 12$. The total surplus with the subsidy is $\pi + \text{CS} - k = 15$. The subsidy increases the total surplus of the market.

2. Consider a monopolist that operates on two. The total demand on the first market is $D_1(p_1) = 80 - p_1$. The total demand on the second market is $D_2(p_2) = 40 - p_2$. Let the cost function be $C(q) = 20q$.

(a) Suppose the monopolist cannot price discriminate , i.e., it faces a single demand $D(p) = D_1(p) + D_2(p)$. Find the optimal price and quantity on each market, total monopoly profit, consumer surplus, and total surplus. The inverse demand is $P(q) = 60 - q/2$, for $q \leq 40$, and $P(q) = 80 - q$ for $q > 40$. There are two cases to consider.

If the monopolist wants a price less than 40, consumers from both markets would make purchases. The profit function would be $\pi(q) = 40q - q^2/2$. First order condition would yield $q = 40$. Price would be $p = 40$. Profits would be $\pi = 800$.

If the monopolist wants a price greater or equal to 40, there would only be a positive demand in the first market. The profit function would be $\pi(q) = 60q - q^2$. First order condition would yield $q = 30$. Price would be $p = 50$. Profits would be $\pi = 900$. Since this case yields higher profits, this is the actual solution chosen by the monopolist.

Quantities consumed are $q_1 = D_1(50) = 30$ and $q_2 = D_2(50) = 0$. Consumer surplus in each market are $\text{CS}_1 = (80 - p)q_1/2 = 450$ and $\text{CS}_2 = 0$. Total surplus is 1350.

(b) Suppose the monopolist can charge a different price on each of the two markets. Find the optimal price and quantity on each market, total monopoly profit, consumer surplus, and total surplus. On market 1, the solution is as before (why?): $q_1 = 30$, $p_1 = 50$, $\pi_1 = 900$, and $\text{CS}_1 = 450$.

On market 2, the profit function is $\pi_2(q_2) = 20q_2 - q_2^2$. First order condition yields $q_2 = 10$. Price is $p_2 = 30$. Profits Are $\pi_2 = 100$. Consumer surplus is $\text{CS}_2 = 50$.

The total surplus is 1500.

(c) Suppose that the monopolist can separate the markets and practice *perfect* price discrimination within each market. Find the quantity traded on each market, total monopoly profit, consumer surplus, and total surplus. The monopolist would produce the efficient quantity on each market. These quantities are $q_1 = D_1(20) = 60$ and $q_2 = 20$. The monopolist would extract all market surplus. Its profits would be $\pi = 1800 + 200 = 2000$.

3. Show that underbidding in a second-price sealed-bid auction is weakly dominated. Fix some player i, and let us compare the utility from bidding v_i vs. the utility from bidding some number $b_i < v_i$. There are three cases to consider, depending on the value of t_i.

Case 1— If $t_i < b_i$, then i would win the auction regardless of whether she bid b_i or v_i. The price paid by i would also be the same in either case. Therefore, i would be indifferent between bidding b_i and v_i.

Case 2— If $t_i > v_i$, then i would lose the auction regardless of whether she bid b_i or v_i, and her utility would be 0 in either case.

Case 3— The only remaining possibility is to have $t_i > b_i$ and $t_i < v_i$. In this case, if i were to bid b_i, she would lose the auction and her utility would be 0. If she were to bid v_i she would win the object and her utility would be $v_i - t_i > 0$. Therefore, she would strictly prefer to bid v_i.

It follows that underbidding is always weakly dominated.

	1	2	3	4	5	6	7
1	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
2	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
3	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
4	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
5	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
6	40, 30	35, 35	30, 40	25, 45	20, 60	35, 35	60, 10
7	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

4. Suppose Henry and George are ice-cream vendors selling the same product at the same price. They choose a location for their vending carts along the beach. Suppose that the beach is divided into 5 uniformly spaced regions. On each region there are 10 people that will buy ice-cream from the closest vendor, splitting evenly if the vendors are at equal distance. Henry and George choose their location simultaneously They make $1 in profits for each costumer they serve.

(a) Write down a 5×5 matrix representing this game. See table.

(b) Execute the iterated dominance algorithm to find all rationalizable locations. See table.

5. Consider two firms selling differentiated varieties of a product, e.g., Coke and Pepsi. Each firm j chooses a price p_j for its own variety. Since these varieties are close substitutes, the demand that each firm faces depends not only on its own price, but also the price of its competitor. Specifically, the demand for j's variety is given by

$$D_j(p_j, p_{-j}) = \max\{0,\, 60 + p_{-j} - 2p_j\}$$

Suppose that both firms can produce any amount of their variety at *no cost*.

(a) Find firm j's best response function. Profits are $\pi_j = p_j(60 + p_{-j} - 2p_j) = (60 + p_{-j})p_j - 2p_j^2$. First order condition yields $\text{BR}_j(p_{-j}) = 15 + p_{-j}/4$.

(b) Assume that firms choose prices simultaneously and independently. Show that choosing $p_j = 18$ is not rationalizable. [*Hint*: perform two rounds of iterated dominance] First round: BR_j only takes values greater than $\text{BR}_j(0) = 15$. Second round: if $p_{-j} \geq 15$, then BR_j only takes values greater than $\text{BR}_j(15) = 18.75$. Therefore, 18 was eliminated in the second round and is thus *not* rationalizable.

(c) Assume that firms choose prices simultaneously and independently. Find the *unique* equilibirum of the game. [*Hint*: symmetric games typically have a symmetric equilibrium] In a symmetric equilibirum $p_j = 15 + p_j/4$. Solving for p_j yields $p_j = 60/3 = 20$.

(d) Assume that firm 1 chooses its price first, and firm 2 chooses its price second after seeing firms 1's price. Find the Stackelberg equilibrium of the game. On the second stage firm 2 will choose $\text{BR}_2(p_1) = 15 + p_1/4$. On the first stage, firm 1 anticipates this and maximizes

$$\pi_1 = (60 + \text{BR}_2(p_1))p_1 - 2p_1^2 = (75 + p_1/4)p_1 - 2p_1^2 = 75p_1 - \frac{7}{4}p_1^2$$

The first order condition yields $p_1 = 150/7$. Therefore, $p_2 = \text{BR}_2(150/7) = 285/14$.

(e) Compare the equilibrium profits of each firm under part (c), withe their profits on part (d). In part (c) firms profits were $\pi = 20D_j(20, 20) = 800$. In part (d) firm profits are $\pi_1 \approx 803.57$ and $\pi_2 \approx 828.83$.

6. Two firms engage in Cournot competition. Each firm j chooses a quantity $q_j \in [0, \infty)$ to supply to the market. These choices are made simultaneously and independently. Both firms have the same cost function $C(q_j) = 10q_j$. The market demand function is given by $D(p) = \max\{0, 100 - p\}$

(a) Find the equilibrium quantity and price, the consumer surplus, and the profits of each firm. Inverse demand is $P(q) = 100 - q$. Profit function is $\pi_j = (100 - q_j - q_{-i})q_j - 10q_j = (90 - q_{-j})q_j - q_j^2$. First order condition yields $\text{BR}_j = 45 - q_{-j}/2$. In the unique equilibrium $q_j = 45 - q_j/2$, and thus $q_j = 90/3 = 30$. Price is $p = P(30+30) = 40$. Profits are $\pi_j = 40 \cdot 30 = 1200$. Consumer surplus is $\text{CS} = (100 - p)q/2 = 1800$.

(b) Now suppose that the government introduces a \$10 tax per unit sold, to be paid by the consumers. Find the equilibrium quantity and price, the consumer surplus, the profits of each firm, and the tax revenue. With the tax $p_b = p_s + 10$. Inverse demand is $P_b(q) = 100 - q$. Profit function is $\pi_j = (P_b(q) - 10)q_j - 10q_j = (80 - q_{-j})q_j - q_j^2$. First order condition yields $\text{BR}_j = 40 - q_{-j}/2$. In the unique equilibrium $q_j = 40 - q_j/2$, and thus $q_j = 80/3 = 26.\bar{6}$. Price paid by the sellers is $p_b = 100 - 160/3 = 140/3 = 46.\bar{6}$ (including the tax paid). Price received by the sellers is $p_s = 36.\bar{6}$. Profits are $\pi_j = 6400/9 = 711.\bar{1}$. Consumer surplus is $\text{CS} = (100 - p)q/2 = 1422.\bar{2}$. Tax revenue is $\text{TR} = 10 * \cdot q = 533.\bar{3}$.

(c) Does this tax result in dead-weight loss? Without the tax, total surplus is $\text{CS} + \pi_1 + \pi_2 = 4200$. With the tax, total surplus including tax revenue is $\text{CS} + \pi_1 + \pi_2 + \text{TR} = 3911.\bar{1}$. Therefore, the tex does increase dead-weight loss.

7. Consider a market with $n \geq 2$ firms engaged in Cournot competition. The firms' cost functions, and the market demand function are as in problem 6.

(a) Find the equilibrium quantity and price, the consumer surplus, and the profits of each firm. Inverse demand is $P(q) = 100 - \sum_i q_i$. Profit function is $\pi_j = (100 - \sum_i q_i)q_j - 10q_j = (90 - \sum_{i \neq j} q_i)q_j - q_j^2$. First order condition yields $\text{BR}_j = 45 - \sum_{i \neq j} q_i/2$. In the unique equilibrium $q_j = 45 - (n-1)q_j/2$, and thus $q_j = 90/(n+1)$. Price is $p = 100 - n90/(n+1)$. Profits are

$$\pi_j = (p - 10)q_j = \left(90 - \frac{n}{n+1}90\right)\frac{90}{n+1} = 8100\frac{1}{(n+1)^2}$$

Consumer surplus is

$$\text{CS} = \frac{1}{2}(100 - p)q = \frac{1}{2}\left(\frac{n90}{n+1}\right)^2 = 4050\left(\frac{n}{n+1}\right)^2$$

(b) How much total surplus is lost due to market power? The maximum total surplus is attained by the competitive equilibrium with $p = 10$ and $q = 90$. The surplus is equal to $(100 - p)q/2 = 4050$. The total surplus lost due to market power is thus

$$\begin{aligned} 4050 - \pi_1 - \pi_2 - \text{CS} &= 4050 - 2 \cdot 8100\frac{1}{(n+1)^2} - 4050\left(\frac{n}{n+1}\right)^2 \\ &= 4050\left(1 - \frac{4}{(n+1)^2} - \frac{n^2}{(n+1)^2}\right) = 4050\frac{2n-2}{(n+1)^2} \end{aligned}$$

(c) What happens to the equilibrium quantity, price, and dead-weight loss when the number of firms grows to infinity? They converge to the competitive quantity, competitive price, and 0, respectively.

8. Suppose two long-lived firms are engaged on *repeated* Cournot competition. The firms have a common discount factor δ. Each period, firms choose simultaneously how much to produce. Suppose that each firm can only choose between a high output (H), or a low output (L). The stage game payoffs are as follows.

	H	L
H	4, 4	1, 6
L	6, 1	2, 2

(a) Which actions are dominated in the stage game? H is dominated by L for each player.

(b) Which outcomes are Pareto dominated in the stage game? (L,L) is Pareto dominated by (H,H)

(c) Find a value in $(0, 1)$ for the discount factor such that the firms can use a grim trigger strategy to implement an efficient outcome. The value from complying is $V(comply) = 4/(1-\delta)$. The maximum value from deviating is $V(deviate) = 6 + \delta 2/(1-\delta)$. We have $V(comply) \geq V(deviate)$ if and only if:

$$\frac{4}{1-\delta} \geq 6 + \frac{2\delta}{1-\delta} \quad \Leftrightarrow \quad 4 \geq 6(1-\delta) + 2\delta = 6 - 4\delta \quad \Leftrightarrow \quad \delta \geq \frac{1}{2}$$