Monopoly

Bruno Salcedo*

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Previously, we studied markets in which there firms act as price takers. This note focuses on the opposite extreme. We consider industries with a single firm dictates the price. Such industries are called *monopolies*, and the corresponding firms are called *monopolists*. These notes briefly cover the models discussed in class. For more context and discussion, see chapters 25 and 26 in Varian (2014).

1. Profit Maximization

Consider a market with a strictly decreasing demand function D served by a single firm—the *monopolist*. Being the only supplier, the monopolist can set the price constrained only by the consumers' willingness to pay. We model this idea by assuming that the monopolist chooses how much to produce (q), and then chooses the highest price (p) that would lead consumers to demand such quantity. This chosen price and quantity must satisfy $D(p) = q$ (why?). Hence, we can find the price by inverting the demand function.

The inverse of demand function is called the *inverse demand*, and I will denote it by $P(\cdot)$. In order to find the inverse demand, you have to write the equation $q = D(p)$, and solve for the price. The formula that you obtain will only be valid over the range of D.

*Department of Economics, Western University · brunosalcedo.com · bsalcedo@uwo.ca

Example 1.1 Suppose that the demand function is given by $D(p) = (1+p)^{-2}$. We can find the inverse demand as follows:

$$q = \left(\frac{1}{1+p}\right)^2 \iff 1+p = \frac{1}{\sqrt{q}} \iff p = \frac{1}{\sqrt{q}} - 1 = \frac{1-\sqrt{q}}{\sqrt{q}}.$$

Hence, the inverse demand function is given by $P(q) = (1-\sqrt{q})/\sqrt{q}$. In this example, the range of D is $(0, 1]$. There is no price that would induce the consumers to demand a quantity greater than one.

Using the inverse demand, we can write the *revenue* (R) of the monopolist as a function of the quantity produced

$$R(q) = qP(q).$$

The *profit* (π) of the monopolist is thus given by

$$\pi(q) = qP(q) - C(q),$$

where $C(q)$ is the economic cost of producing quantity q. The monopolist chooses $q \geq 0$ to maximize $\pi(q)$. The solution to this problem is called the *monopolistic quantity* and is denoted by q^M. Under some regularity conditions that are beyond the scope of this class, the monopolistic quantity is the unique solution to the first-order condition

$$\text{MR}(q) = \text{MC}(q), \tag{1}$$

where $\text{MR}(\,\cdot\,)$ and $\text{MC}(\,\cdot\,)$ denote the monopolist's *marginal revenue* and *marginal cost*, respectively. Once you find the monopolistic quantity, you can find the *monopolistic price* (p^M) using the inverse demand.

Example 1.2 Suppose that the demand is as in Example 1.1, and the monopolist's cost function is given by $C(q) = c \cdot q$, for some constant $c > 0$. Note that we have $\text{MC}(q) = c$ for all q. The revenue function is given by

$$R(q) = q \cdot \frac{1-\sqrt{q}}{\sqrt{q}} = \sqrt{q}(1-\sqrt{q})$$

Taking derivatives,

$$\text{MR}(q) = -\sqrt{q} \cdot \frac{1}{2\sqrt{q}} + (1 - \sqrt{q}) \cdot \frac{1}{2\sqrt{q}} = \frac{1 - 2\sqrt{q}}{2\sqrt{q}}$$

The first-order condition (1) requires marginal revenue to equal marginal cost,

$$\frac{1 - 2\sqrt{q}}{2\sqrt{q}} = c \implies 1 - 2\sqrt{q} = 2c\sqrt{q} \implies q^M = \left(\frac{1}{2c+2}\right)^2.$$

The inverse demand then gives us the monopolistic price

$$p^M = P\left(\frac{1}{(2c+2)^2}\right) = \frac{1 - 1/(2c+2)}{1/(2c+2)} = \frac{2c+2-1}{1} = 2c+1.$$

Example 1.3 Suppose that the market demand is $D(p) = 120 - 2p$, and the monopolist's cost function is $C(q) = 20 + q^2/4$. The inverse demand is given by

$$q = 120 - 2P(q) \implies P(q) = 60 - \frac{1}{2}q.$$

The revenue function is thus

$$R(q) = q\left(60 - \frac{1}{2}q\right) = 60q - \frac{1}{2}q^2.$$

Taking derivatives of the cost and revenue functions yields

$$\text{MR}(q) = 60 - q \quad \text{and} \quad \text{MC}(q) = \frac{1}{2}q.$$

The first-order condition is thus

$$60 - q^M = \frac{1}{2}q^M \implies 3q^M = 120 \implies q^M = 40.$$

The monopolistic price is thus

$$p^M = P(20) = 60 - \frac{40}{2} = 40.$$

The monopolistic profits are

$$\pi = 40 \cdot 40 - \left(20 + \frac{1}{4}20^2\right) = 1480.$$

2. Markup Pricing

Note that we can express marginal revenue as follows

$$\begin{aligned}\mathrm{MR}(q) &= \frac{d}{dq}\big(qP(q)\big) = P(q) + q\frac{dP(q)}{dq} = P(q)\left(1 + \frac{q}{P(q)} \cdot \frac{dP(q)}{dq}\right) \\ &= P(q)\left(1 + \frac{1}{p/D(p)} \cdot \frac{1}{dD(p)/dp}\right) = P(q)\left(1 + \frac{1}{e}\right),\end{aligned}$$

where e is the price-elasticity of demand. Substituting with this expression in (1), we can rewrite the first-order condition for the monopolist as follows

$$\frac{P(q)}{\mathrm{MC}(q)} = \frac{1}{1 + 1/e} \tag{2}$$

The ratio of price to marginal cost on the left-hand side of equation (2) is sometimes called the *markup*. It measures how much the monopolist sets prices above the marginal cost.

Example 2.1 We can verify equation (2) revisiting Example 1.2. Note that

$$D'(p) = -\frac{2}{(1+p)^3},$$

and therefore the elasticity of demand is given by

$$e = -\frac{2}{(1+p)^3} \cdot \frac{p}{(1+p)^{-2}} = -\frac{2p}{1+p}.$$

Using $\mathrm{MC}(q) = c$, (2) thus implies that the monopolistic price is the solution to

$$\frac{p}{c} = \frac{1}{1 - (1+p)/2p} = \frac{2p}{2p - (1+p)} = \frac{2p}{p-1} \implies p^M = 2c + 1,$$

which is the same solution we found before. In situations in which the elasticity of demand is easier to compute, Equation (2) could be the best way to find the monopolistic price.

Equation (2) is an important equation that teaches us different things:

1. The only information needed in order to find the profit-maximizing price is

the elasticity of demand. This is part of the reason why economists devote so much effort in finding ways to estimate demand elasticities.

2. Since demand is decreasing, the elasticity of demand is negative. Hence, in order for the price to be positive, we need

$$1 + \frac{1}{e} > 0 \implies 1 > -\frac{1}{e} \implies |e| = -e > 1.$$

This tells us that monopolists will always choose to operate in the elastic part of the demand function.

3. Since $e < 0$ and $|e| > 1$, we have that $1 + 1/e < 1$ and thus

$$\frac{1}{1 + 1/e} > 1.$$

Hence, equation (2) implies that the monopolist always sets the price to be greater than the marginal cost. In contrast, in competitive markets the price equals the marginal cost of the firms. The monopolist uses its market power to charge high prices.

4. Suppose that we compare two markets with demand elasticities e_1 and e_2 such that $|e_1| > |e_2| > 1$. Then,

$$e_1 < e_2 < 0 \implies 1 + \frac{1}{e_1} < 1 + \frac{1}{e_2} \implies \frac{1}{1 + 1/e_1} > \frac{1}{1 + 1/e_2}.$$

Equation (2) thus implies that monopolists set higher prices in less elastic markets.

3. Regulating a Monopoly

Monopolies are very different from competitive markets. In particular, we have shown that the monopolistic price is greater than the competitive price. Since the competitive price is the only fixed-price mechanism that maximizes surplus, it follows that monopolies are inefficient. Because of this, it is possible for taxes,

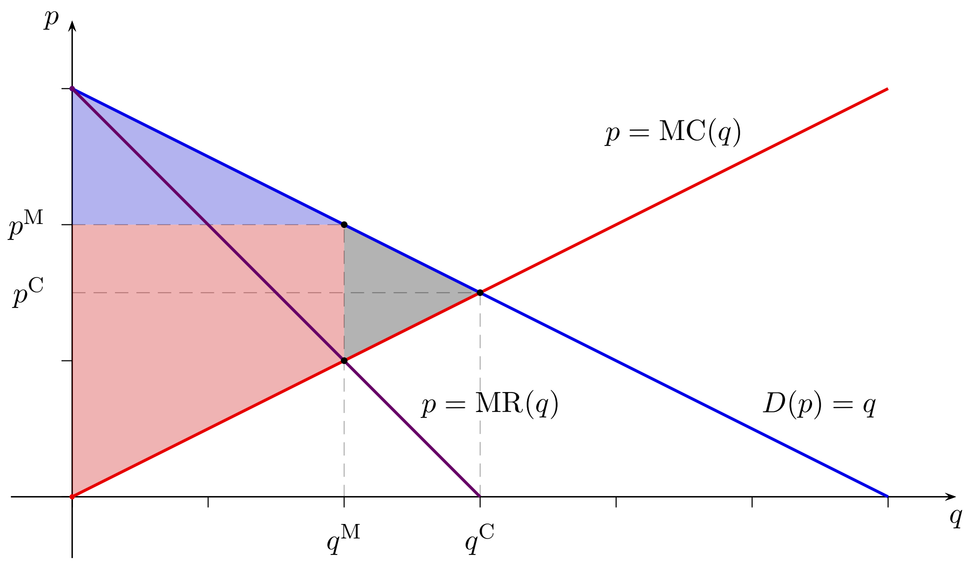

Figure 1 – Monopoly vs. competitive market with demand $D(p) = 120 - 2p$ and cost $C(q) = 20 + q^2/4$.

price-controls, and other government policies to improve efficiency. However, because of time limitations, we did not discuss them in class. If you are interested in this topic you can read Chapter 25 in Varian (2014).

Example 3.1 Let us revisit Example 1.3 and compare the monopolistic outcome with the outcome of a competitive market with the same cost and demand functions. The competitive supply function $S(\,\cdot\,)$ is given by

$$\mathrm{MC}(S(p)) = p \implies \frac{1}{2}S(p) = p \implies S(p) = 2p.$$

The competitive price (p^C) can be derived from the market clearing condition

$$S(p) = D(p) \implies 2p = 120 - 2p \implies p^C = 30.$$

The competitive quantity (q^C) is thus given by

$$q^C = S\,(30) = 2 \cdot 30 = 60.$$

Hence, we can see that $p^M > p^C$ and $q^M < q^C$, as expected. This results in a deadweight loss given by the area of the gray triangle in Figure 1.

$$\text{DWL} = \frac{1}{2}\left(p^M - MC\left(q^M\right)\right)\left(q^C - q^M\right) = \frac{1}{2}(40-20)(60-40) = 200.$$

4. Price Discrimination

So far, we have considered a very narrow set of choices for the monopolist. We have only allowed it to choose a quantity and a fixed price per unit sold. In reality, firms use much more complicated pricing schemes in which the price can depend on either the quantity bought, the identity or characteristics of the buyer, or both. This is called *price discrimination* and can have important consequences both for the monopolist's profits, and for the degree of deadweight loss.

For simplicity, we will assume throughout this section that the monopolist's cost function can be written as $C(q) = c \cdot q$, for some constant $c > 0$. This assumption guarantees that the marginal cost of the monopolist is constant, $\text{MC}(q) = c$. We could derive similar conclusions in more general settings without marginal costs, but the analysis would be significantly more complicated.

4.1. Segmented Markets

4.1.a. *Unit Demand.–* Consider a market with n consumers. Suppose that the monopolist can charge a different price p_i to each consumer. Further suppose that the demand of consumer i can be written as

$$D_i(p) = \begin{cases} 1 & \text{if} \quad p \leq v_i \\ 0 & \text{if} \quad p > v_i \end{cases},$$

where $v_i > 0$ is a fixed parameter that captures i's willingness to pay. This implies that each consumer i desires to consume *at most* one unit of the product.

For example, the monopolist could be a landlord renting apartments. Most potential tenants will typically only need at most one apartment. The key assumption is that the landlord can price discriminate between different potential

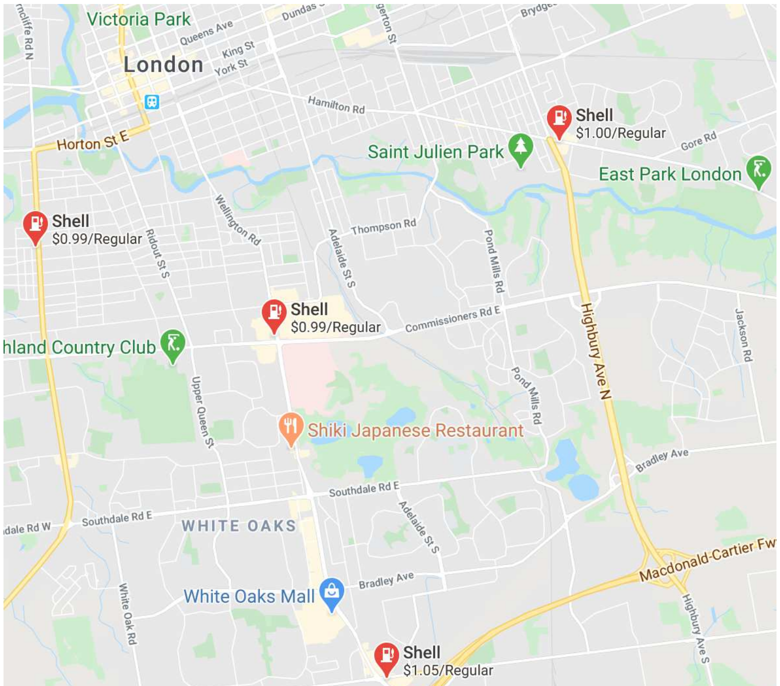

Figure 2 – Price of gas at different Shell stations near London, ON (Google, 2020).

tenants. This could happen, for instance, if there is a high listing price but the landlord can offer a different discount to each potential tenant.

There are two cases to consider. If $v_i < c$, then the cost of producing one additional unit of the object is greater than the consumer's willingness to pay. In this case, the monopolist would not sell the object to the consumer. Otherwise, if $v_i \geq c$, then the monopolist will set the price for consumer i to be exaclty equal to their willingness to pay $p_i = v_i$. Note that this outcome maximizes total surplus (there is no deadweight loss), but all the surplus goes to the monopolist (consumer surplus is equal to zero).

4.1.b. *General Demand.*– Consider a monopolist can charge different prices on different markets or, equivalently, to different consumers. For example, Shell owns different gas stations in town, including one on Wellington Rd, right next to the 401 highway. You can think of each gas station as a *local monopoly* for those consumers who are nearby and want to buy gas. Shell charges slightly different prices on different gas stations. Consumers on the highway typically exhibit a less elastic demand. Hence, the gas stations that are close to a highway tend to charge higher prices. See Figure 2.

Suppose that there are two markets with strictly increasing demand functions D_1 and D_2, respectively. The monopolist can charge different prices p_1 and p_2 on each market. As before, the revenue on market i can be expressed in terms of the inverse demand as

$$R_i(q_i) = q_i P_i(q_i),$$

for $i = 1, 2$. The total profit of the monopolist is given by

$$\pi(q_1, q_2) = q_1 P_1(q_1) + q_2 P_2(q_2) - C(q_1 + q_2).$$

Note that the cost of the monopolist depends on the total amount it produces for all consumers. The monopolist chooses $q_1 \geq 0$ and $q_2 \geq 0$ to maximize their profit. As before, the solution can often be found using the first-order conditions

$$\mathrm{MR}_1(q_1) = \mathrm{MC}(q_1 + q_2) = \mathrm{MR}_2(q_2).$$

As part of the homework, you are asked to solve a specific example. You are also asked to compare the outcome of a monopolist that can price discriminate across two different markets, and a monopolist who has to set the same price in both markets. If you solve the problem correctly, you should see that the ability to price discriminate increases the profit of the monopolist and reduces deadweight loss. This is true in general. In the specific example, consumer surplus also goes up, but this need not be the case in general.

4.2. Non-Linear Pricing

Starbucks charges different unit prices depending on how much coffee people buy. If you buy a small cappuccino, you pay \$3.95 for 12oz, or \$0.33 per ounce. If you buy a medium cappuccino, you pay \$4.45 for 16oz, or \$0.28 per ounce. If you buy a large cappuccino, you pay \$4.95 for 20oz, or \$0.25 per ounce. The more you buy, the lower it is the price you pay for each unit bought. This kind of bulk discounting is a form of *non-linear pricing*.

4.3. Perfect Price Discrimination

We have thus far allowed to price discriminate on either the identity of the buyer, or the quantity bought, but not both. Now, suppose that the monopolist can price discriminate both based on identity and quantity. This form of price discrimination is called *perfect price discrimination*.

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