Oligopoly

Bruno Salcedo*

Winter 2020

An oligopoly is market operated by *a few* firms. The term "a few" is to be interpreted as follows. First, it means that there is more than one firm. Second, it also means that there are *not* that many firms so that all firms act as price takers. At least some firms know that they can have a significant effect on the market price. Oligopolies cannot be analyzed using our perfect competition model, nor our monopoly model. These notes provide some models that are better suited to capture the strategic aspects of oligopolistic markets.

1. Benchmark models

All the models in these notes are presented in the context of a specific market. The total market demand is given by:

$$D(p) = 20 - 2p. \tag{1}$$

All firms operating in this market have the same cost function given by:

$$C(q) = 2q. \tag{2}$$

As a benchmark, let us start by solving our competitive and monopolistic models in the context of this market.

*Department of Economics, Western University · brunosalcedo.com · bsalcedo@uwo.ca

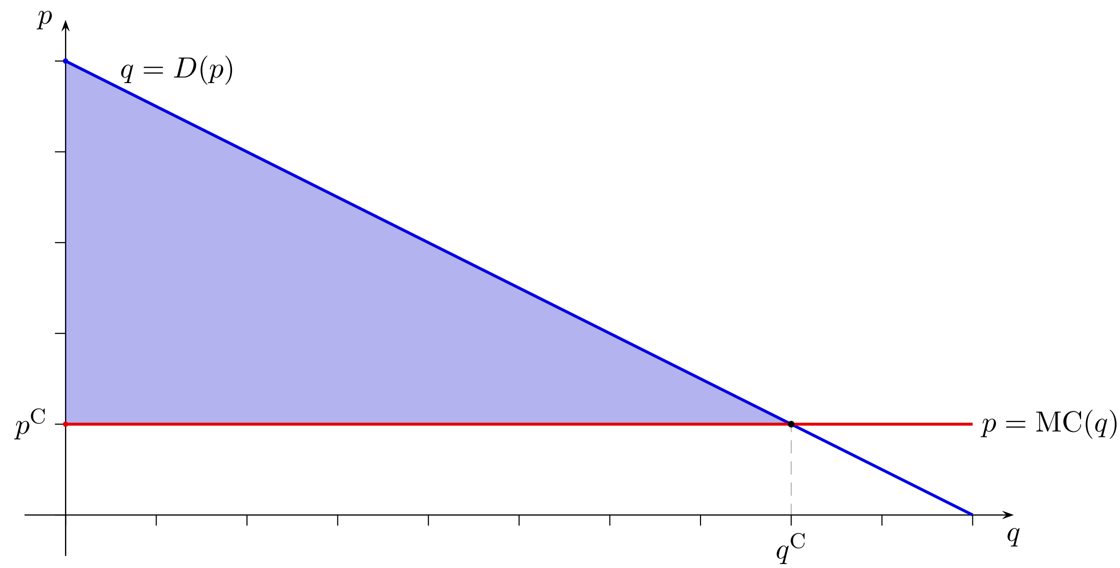

Figure 1 – Competitive equilibrium

Suppose that all firms are price takers. The market equilibrium would be determined by the intersection of supply and demand. See Figure 1. The equilibrium quantity would be $q^{\mathrm{C}} = 16$ and the equilibrium price would be $p^{\mathrm{C}} = 2$. Firms would make zero profits (this is because of the constant marginal cost). And the total market surplus would be the area of the blue triangle in the figure, $8 \times 16/2 = 64$.

Suppose instead that the market is operated by a monopolist. To solve the monopolistic problem, we need the inverse market demand, which is given by:

$$P(q) = 10 - \frac{q}{2}. \tag{3}$$

The monopolist's profits are given by

$$\pi(q) = qP(q) - C(q) = 8q - \frac{1}{2}q^2. \tag{4}$$

The optimal quantity chosen by the monopolist is given by the first-order condition: $8 - q^{\mathrm{M}} = 0$. Consequently, the monopolist would produce $q^{\mathrm{M}} = 8$. The corresponding market price is given by the inverse demand function $p^{\mathrm{M}} = P(q^{\mathrm{M}}) = 6$. The monopolist profits would be $8 \times (6 - 2) = 32$. This would result in a deadweight loss of 16. See Figure 2.

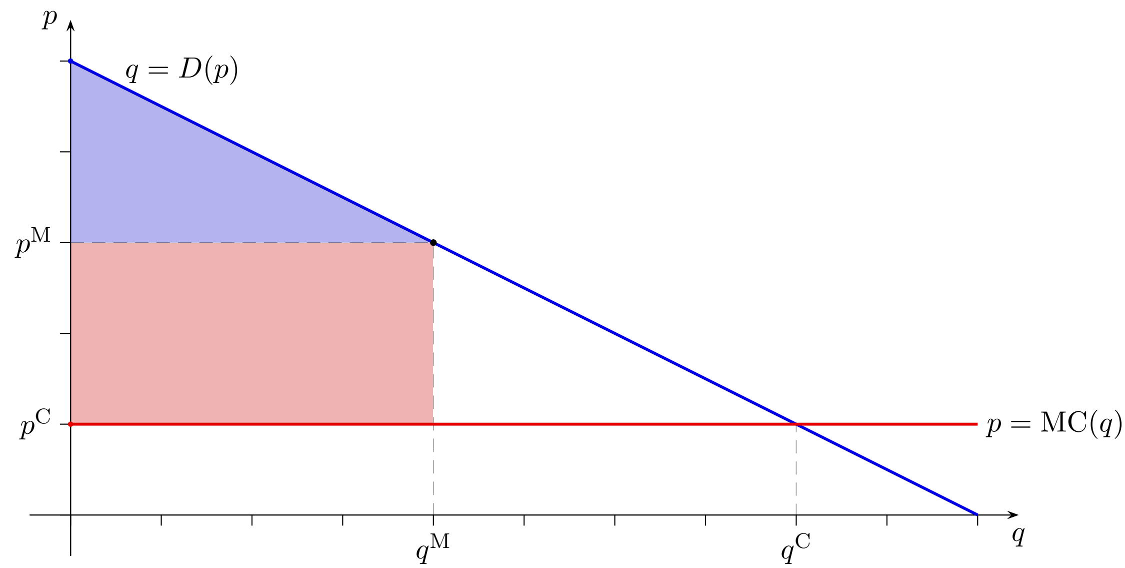

Figure 2 – Monopoly

2. Cournot duopoly

Now suppose that there are two firms, Firm 1 and Firm 2. Each firm chooses how much it produces. These choices are made independently and simultaneously. This model is called *Cournot duopoly*. The quantity produced by firm j is denoted by $q_j \geq 0$. The price at which firms can sell their production is determined by the inverse demand function and the total production. That is, $p = P(q_1 + q_2)$. The revenue of each firm equals the market price times the quantity sold by the firm. Firm j's profit function is thus given by:

$$\pi_j = q_j P(q_j + q_{-j}) - C(q_j) = \left(8 - \frac{1}{2} q_{-j}\right) q_j - \frac{1}{2} q_j^2. \tag{5}$$

Let us compare this profit function with that of a monopolist, given in equation (4). The difference is in the first term. The duopolist faces a smaller market size, because part of the market is serviced by another firm. The duopolist optimal choice depends on the quantity produced by its competitor. This optimal choice is characterized by the first order condition $(8 - q_{-j}/2) - q_j = 0$. Firm i's best response function is thus

$$\mathrm{BR}_i(q_{-i}) = 8 - \frac{1}{2} q_{-j}. \tag{6}$$

2.1. Rationalizability

Equation (6) is not enough to predict the output when there are two firms, but it is a start. Note that the firms' best response functions never take values greater than 8. See the first panel of Figure 3. In fact, we could show (but we won't), that every quantity greater than 8 is strictly dominated. Rational firm would never choose quantities greater than 8.

Now suppose that Firm 2 know that Firm 1 is rational. Then, Firm 2 would know that q_1 would be between 0 and 8. Firm 2's best responses would thus be between $\text{BR}_2(8) = 4$ and $\text{BR}_2(0) = 8$. See the second panel of Figure 3. If there is mutual knowledge of rationality, then each firms will produce a quantity between 4 and 8.

Now suppose that Firm 1 knows that Firm 2 knows that Firm 1 is rational. Then, Firm 2 would know that q_1 would be between 4 and 8. Firm 1's best responses would thus be between $\text{BR}_2(8) = 4$ and $\text{BR}_2(4) = 6$. See the third panel of Figure 3. If there is 2^{nd} mutual knowledge of rationality, then each firms will produce a quantity between 4 and 6.

We can continue this process. If there is 3^{rd} mutual knowledge of rationality, then each firms will produce a quantity between $\text{BR}_i(6) = 5$ and $\text{BR}_i(4) = 6$. If there is 4^{th} mutual knowledge of rationality, then each firms will produce a quantity between $\text{BR}_i(6) = 5$ and $\text{BR}_i(5) = 5.5$. If there is 5^{th} mutual knowledge of rationality, then each firms will produce a quantity between $\text{BR}_i(5.5) = 5.25$ and $\text{BR}_i(5) = 5.5$. The upper and lower bounds approach one another. In the limit, they coincide, which means that only one quantity survives. See the fourth panel of Figure 3.

Claim 1 *In Cournot duopolies, the only rationalizable outcome is the intersection of the best response functions.*

2.2. Cournot equilibrium

The intersection of the best response functions is called the *Cournot equilibrium.* In general Cournot competition models, it can be found by solving a system of n variables and n equations of the form $q_i = \text{BR}_i(q_{-i})$, where n is the number of firms. When all firms have exactly the same utility function the equilibrium

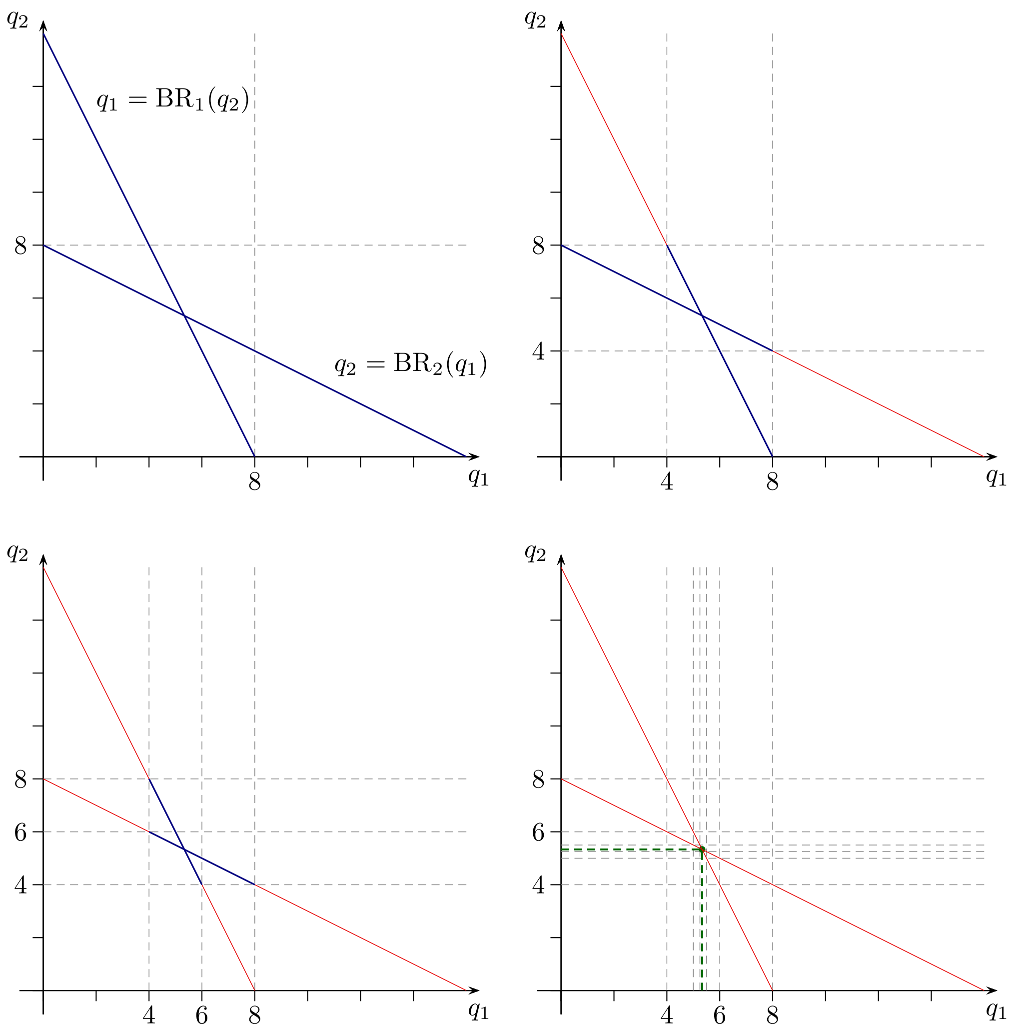

Figure 3 – Iterated dominance in Corunot competition

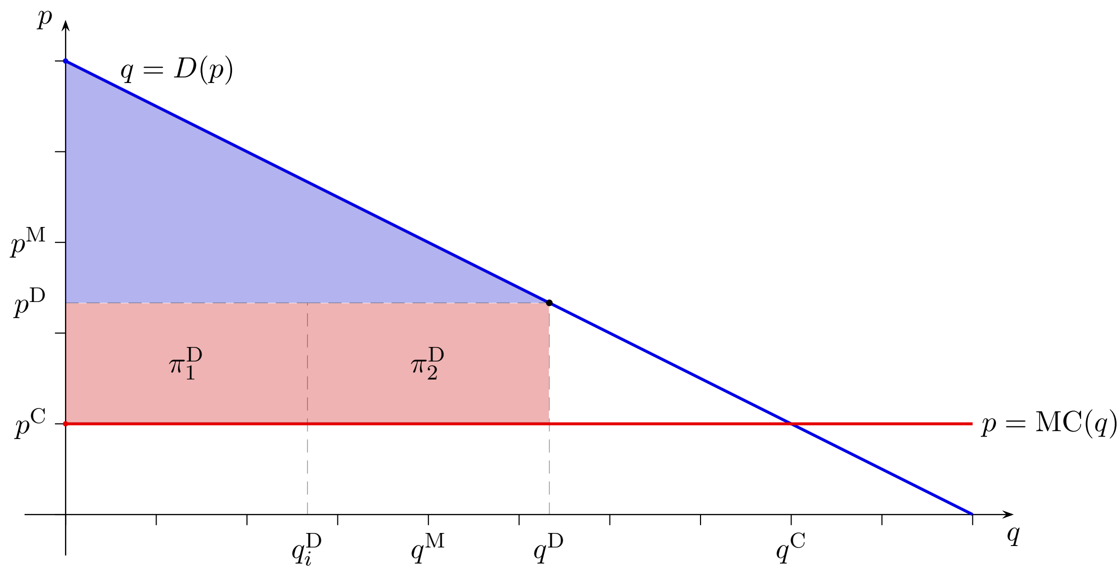

Figure 4 – Cournot

	price	quantity	profits	surplus	dwl
Competitive	2	16	0	64	0
Monopoly	6	8	32	48	16
Cournot	$4.\bar{6}$	$10.\bar{6}$	$28.\bar{4}$	$28.\bar{4}$	$7.\bar{1}$

Table 1 – Comparison of different market arrangements

is symmetric, in that all firms produce the same quantity. In such cases, we can find the Cournot equilibrium by solving a single equation $q_i^{\mathrm{D}} = \mathrm{BR}_i(q_i^{\mathrm{D}})$.

Our example is symmetric. Using equation (6) we get $q_i^{\mathrm{D}} = 8 - q_i^{\mathrm{D}}/2$. Hence, if there is common knowledge of rationality, both firms would produce $q_i^{\mathrm{D}} = 16/3 = 5.\bar{3}$. The total quantity produced would be $q^{\mathrm{D}} = 2q_i^{\mathrm{D}} = 32/3$. The corresponding market price is given by the inverse demand function $p^{\mathrm{D}} = P(q^{\mathrm{D}}) = 14/3 = 4.\bar{6}$. The profits of *each* firm would be $(16/3) \times (14/3 - 2) = 128/9$. The total dead-weight loss would be $64/9$. See Figure 4.

Note that the Cournot equilibrium is in the middle ground between perfect competition and a monopoly. This is in terms of price, quantity, firm profits, and total market surplus. See table. The same is true about most models of oligopolistic competition. In the problem set you are asked to show that, when the number of firms in an oligopoly grows, the outcome gets closer to that of a competitive market.

3. Stackelberg leadership

The model of Cournot competition assumes that firms in the oligopoly make their choices simultaneously and independently. That is not always the case. In reality, firms choices are often asynchronous, and the dynamic structure of choices can affect the equilibrium outcome.

Suppose for instance that firms make choices sequentially as follows. First, firm 1 chooses how much to produce and firm 2 observes q_2. Afterwards, firm 2 chooses how much to produce. This dynamic structure is called *Stackelberg leadership*. The first firm to choose is called the *leader*, and the second one the *follower*. As we shall see, the outcome of this game is different from the Cournot equilibrium.

The way to solve this dynamic game is via *backwards induction*. The follower is rational, and observes q_1 before choosing q_2. Hence, it will choose the quantity that maximizes its profits given q_1, that is, it will choose to produce $q_2 = \mathrm{BR}_2(q_1)$. Note that this optimal choice depends on the quantity chosen by the leader.

The leader knows that the follower is rational, and thus anticipates this dependence. Instead of taking the choice of the follower as given, the leader treats the choice of the follower as a function of its own. The relevant profit function for the leader is thus

$$\pi_1 = q_1 P\Big(q_1 + \mathrm{BR}_2(q_1)\Big) - C(q_1) \tag{7}$$

Substituting with (2), (3) and (6) yields

$$\pi_1 = q_1 \left(10 - \frac{1}{2}\left(q_1 + 8 - \frac{1}{2}q_1\right)\right) - 2q_1 = 4q_1 - \frac{1}{4}q_1^2. \tag{8}$$

The optimal quantity chosen by the leader is given by the first-order condition: $4 - q_1^{\mathrm{S}}/2 = 0$. Consequently, the monopolist would produce $q_1^{\mathrm{S}} = 8$. The follower would produce $q_2^{\mathrm{S}} = \mathrm{BR}_2(q_1^{\mathrm{S}}) = 4$. The corresponding market price is given by the inverse demand function $p^{\mathrm{S}} = P(q_1^{\mathrm{S}} + q_2^{\mathrm{S}}) = 4$. The profits would be $\pi_1^{\mathrm{S}} = 8 \times (4-2) = 16$ and $\pi_2^{\mathrm{S}} = 4 \times (4-2) = 8$.

Claim 2 *The leader in a Stackelberg quantity duopoly makes higher profits than in would in a Cournot duopoly with the same demand and cost functions.*

4. Collusion

In a Cournot duopoly, the total industry profits are less than in a monopoly. See Table 1. If firms could agree to each produce half of the monopolistic quantity each, they could each make half the monopolistic profit. Both firms would be strictly better off than under the Cournot equilibrium. However, this agreement would not be incentive compatible. If we ignore consumer welfare and analyze an oligopoly exclusively from the perspective of the firms, the situation is a social dilemma. When each firm tries to maximize their profits independently, they end up in a situation in which each firm is making suboptimal profits.

We can capture this idea with a much simpler model in which firms can only choose between a high and a low quantities. The corresponding profits are given in Figure 5. If both firms choose a low quantity, the market price will be high and the firms will share high profits. However, if a firm deviates by increasing its quantity, it will enjoy higher profits resulting from high volume sales at a high price. The game has the structure of a prisoners' dilemma. Choosing H is a dominant strategy, but the outcome (H,H) is Pareto dominated (from the perspective of the firms) by the outcome (L,L).

	H	L
H	$20, 20$	$5, 60$
L	$60, 5$	$10, 10$

Figure 5 – Simplified duopoly model

Firms often try different mechanisms to coordinate their choices in order to escape this social dilemma. Any attempt from part of the firms to do so is called *collusion*. Collusion helps to increase the firms profits by bringing the market outcome closer to the monopolistic outcome. This comes at the cost of dead-weight loss. Once we take consumer welfare into account, collusion is inefficient for society. Hence, there are anti-collusion laws in most countries in the world. However, these laws are difficult to implement and firms often collude despite it being illegal.

One possible collusion mechanism is to exploit the long-lasting nature of oligopolistic competition. Most firms are long-lived and interact repeatedly over a series of periods. Coke and Pepsi have competed with one another for over a hundred years, since 1898, and will probably continue to do so for a very long time. When agents interact repeatedly, they can use the promise of future *reciprocity* to generate incentives for cooperation. Each firm might be willing to produce a low quantity today in exchange for the promise that the other firm will continue to produce low quantities in the future, thus securing high profits for both firms.

4.1. Discounted present value

In order to formally model that idea, we need to understand how firms value streams of cash flows over time. Assume that firms have access to a financial institution where they can invest money and received a constant risk-free interest rate $r > 0$. If a firm invested x dollars at time $t = 0$, it would receive $(1 + r)x$ dollars the next period. If it reinvested both the initial investment and the first period return, it would have $(1 + r)^2 x$ dollars at $t = 2$. Similarly, if it continue to reinvest for t consecutive periods, it would receive $(1 + r)^t x$. Hence, if a firm wanted to have y dollars at period t, it would have to invest

$$x = \delta^t y \tag{9}$$

dollars at period $t = 0$, where δ is the number given by $\delta := 1/(1 + r)$. The number δ is called the *discount factor*. Equation (9) allows us to compare the value of money in the future, with the value of money today.

Definition 1 The *discounted present value* of y dollars in period t is $\delta^t y$.

Note that the discount factor δ is always between 0 and 1. Suppose that we want to compute the present value of receiving y on period t, and let us consider two extreme cases. If $\delta = 1$, then the present value would be equal to $1^t y = y$. In that case, the value of money does not depend when it is received. The DM is very *patient*, in that it does not mind waiting to receive a payment far ahead into the future. If $\delta = 0$, then the present value would be $0^t y = 0$ if $t \geq 1$. In that case, the DM does*not* value receiving money in the future. DM is ver y *impatient* in that it only values money if it receives it right now. In general, the discount factor can be interpreted as a measure of patience. High discount factors mean that the DM values the future almost as much as the present, and low discount factors means that the DM cares much more about the present.

Suppose that a firm expects to receive x_0 dollars today, x_1 dollars tomorrow, x_2 dollars the period after tomorrow, x_t dollars on period t, and so on and so forth. This is called a stream of cash flows. One way to assign a value to the whole stream, is to compute the discounted present value of the cash flow in each period, and add these values up.

Definition 2 The *discounted present value* of a stream of cash flows $x_1, x_2, x_3, \ldots$ is given by

$$v = \sum_t \delta^t x_t. \tag{10}$$

For example, suppose that a firm expects to receive x dollars each and every period forever after. The discounted present value of this stream of cash flows would be

$$v = x + \delta x + \delta^2 x + \delta^3 x + \delta^4 x + \ldots. \tag{11}$$

Let us multiply both sides of equation (11) by the discount factor to get

$$\delta v = \delta x + \delta^2 x + \delta^3 x + \delta^4 x + \delta^5 x + \ldots. \tag{12}$$

We can subtract equation (12) from (11). When we do so, a lot of terms cancel out on the right hand side and we are left with

$$v - \delta v = x + \cancelto{0}{\lim_{t\to\infty} \delta^t x} \qquad \Rightarrow \qquad v = \frac{x}{1-\delta}. \tag{13}$$

This is a useful formula to compute the present discounted value of a stream of *constant* cash flows.

4.2. Grim-trigger strategies

Suppose that the firms agree to the following dynamic strategies

- Both firms will produce the low quantity as long as no-one has deviated from the agreement
- If someone ever deviates from the agreement at least once, then both firms will proceed to produce high quantities forever after.

This kind of strategies are called *grim trigger* strategies. We know that, if the firms played the game a single period, H is the dominant action for each firm and this agreement wouldn't work. What if firms where in a long-term relation?

Suppose that firm j expects its competitor to stick to the agreement. If firm j were to also comply with the agreement, then both firms would make a profit of 20 every period for ever after. The present discounted value of this stream of profits is

$$v(\text{comply}) = \frac{20}{1-\delta}. \tag{14}$$

Instead, suppose that firm j deviates at some period. That period, firm j would make a profit of 60. However, in every subsequent period, j's competitor would choose H, which means that j's profits would be at most 10. The present discounted value from deviating is thus

$$\begin{aligned} v(\text{deviate}) &= 60 + \delta 10 + \delta^2 10 + \delta^3 10 + \ldots \\ &= 60 + \delta\left(10 + \delta 10 + \delta^2 10 + \ldots\right) = 60 + \frac{\delta}{1-\delta}10. \end{aligned} \tag{15}$$

The agreement is *incentive compatible* if the value from complying is no less than the value from defecting. Using (14) and (15), the agreement is incentive compatible if

$$\begin{aligned} \frac{20}{1-\delta} \geq 60 + \frac{\delta}{1-\delta}10 \quad &\Leftrightarrow \quad 20 \geq 60(1-\delta) + 10\delta = 60 - 50\delta \\ &\Leftrightarrow \quad \delta \geq \frac{4}{5}. \end{aligned} \tag{16}$$

That is, only if the firms are patient enough so that the temptation of making a short gain at the expense of the other firm does not trump over the value of maintaining a profitable long run relation.