ECON306 – Quiz 2

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There are 4 questions. You have 40min to answer all of them. Don't forget to write your name and PSU ID (e.g. bxs5142) on all the pages that you want to be graded.

1. [30pts] Consider the following model:

$$\text{WAGE}_i = \beta_0 + \beta_1 \text{EXP}_i + \beta_2 \text{EDU}_i + \beta_3 \text{GEND}_i + \varepsilon_i$$

where:

$$\begin{aligned} \text{WAGE}_i &= \text{ the wage of the } i\text{th worker} \\ \text{EXP}_i &= \text{ the years of work experience of the } i\text{th worker} \\ \text{EDU}_i &= \text{ the years of post-high school education of the } i\text{th worker} \\ \text{GEND}_i &= \text{ the gender of the } i\text{th worker (1 for male and 0 for female)} \end{aligned}$$

(a) According to the model, what is the average wage of a male with 5 years of post-high school education and 4 years of experience?

$$\mathbb{E}\left[\text{WAGE}_i \,\middle|\, \text{exp}_i = 4, \text{EDU}_i = 5, \text{GEND}_i = 1\right] = \beta_0 + 4\beta_1 + 5\beta_2 + \beta_3$$

(b) Taking into account your gender and work experience, how much would you expect your future wage to change if you decide to do a 2 year masters after you finish college?

$$\Delta\mathbb{E}[\text{WAGE}_i] = \Delta\text{EDU}_i \times \beta_2 = 2\beta_2$$

(c) What sign would you expect β_1 and β_2 to have?

They should be positive as people with more education and more experience tend to have higher incomes.

(d) [Bonus] If we estimate that $\beta_2 > 0$, can we conclude that more education causes higher wages? Briefly justify your answer (2-4 sentences).

No, because correlation does not imply causation. For example, it might be the case that more productive people choose (or have opportunities) to engage in more years of education. As a result one may observe than people with more education receive higher wages on average, regardless of whether there is a causal relation.

2. [30 pts] Consider the following model for the fertility and income:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

$$\begin{aligned} y_i &= \text{ Fertility rate for the } i^{\text{th}} \text{ state} \\ x_i &= \text{ Logarithm of the per capita income of the } i^{\text{th}} \text{ state} \end{aligned}$$

Use the data from table (1) to answer the following questions.

(a) What are the OLS *estimates* for β_0 and β_1?

$$\hat{\beta}_1 = \frac{\sum(y_i - \bar{y})(x_i - \bar{x})}{\sum(x_i - \bar{x})^2} = \frac{-1.18}{1.04} \approx -1.14$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \approx 2.43 - (-1.14) \times 3.83 \approx 6.81$$

(b) According to the estimated model, what is the average fertility rate for states with $x_i = 2.00$?

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i \approx 6.81 - 1.14 \times \approx 4.52$$

(c) What is the R^2 of the estimated model?

$$R^2 = 1 - \frac{\text{RSS}}{\text{TSS}} = 1 - \frac{\sum e_i^2}{\sum(y_i - \bar{y})^2} = 1 - \frac{1.98}{3.32} \approx 0.40$$

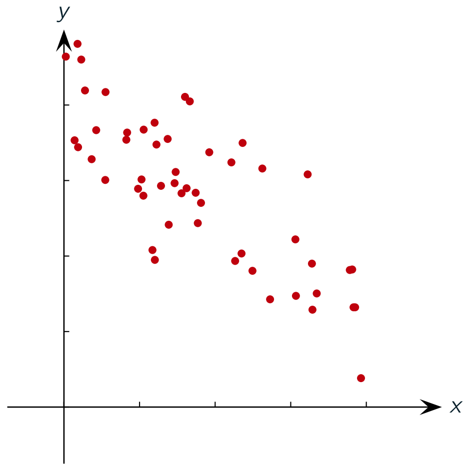
	y_i	x_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$	$(y_i - \bar{y})(x_i - \bar{x})$	$\hat{y}_i$	e_i	e_i^2
1	2.61	3.98	0.14	0.18	0.02	0.03	0.03	2.27	-0.34	0.11
2	2.15	3.98	0.15	-0.28	0.02	0.08	-0.04	2.26	0.11	0.01
3	2.10	3.94	0.11	-0.33	0.01	0.11	-0.04	2.31	0.21	0.04
4	2.39	4.12	0.29	-0.04	0.08	0.00	-0.01	2.11	-0.28	0.08
5	2.11	4.03	0.20	-0.32	0.04	0.10	-0.06	2.20	0.09	0.01
6	2.26	3.91	0.07	-0.17	0.01	0.03	-0.01	2.35	0.09	0.01
7	2.94	3.52	-0.31	0.51	0.10	0.26	-0.16	2.79	-0.15	0.02
8	2.20	4.01	0.18	-0.23	0.03	0.05	-0.04	2.23	0.03	0.00
9	1.80	4.25	0.42	-0.63	0.17	0.40	-0.26	1.96	0.16	0.03
10	2.65	3.83	-0.01	0.22	0.00	0.05	0.00	2.44	-0.21	0.04
11	2.75	3.73	-0.10	0.32	0.01	0.10	-0.03	2.55	-0.20	0.04
12	3.03	3.61	-0.22	0.60	0.05	0.36	-0.13	2.68	-0.35	0.12
13	2.60	3.67	-0.16	0.17	0.03	0.03	-0.03	2.62	0.02	0.00
14	2.51	3.87	0.04	0.08	0.00	0.01	0.00	2.39	-0.12	0.01
15	2.18	3.75	-0.08	-0.25	0.01	0.06	0.02	2.52	0.34	0.12
16	2.80	3.68	-0.15	0.37	0.02	0.13	-0.06	2.61	-0.19	0.04
17	2.10	3.83	0.00	-0.33	0.00	0.11	0.00	2.43	0.33	0.11
18	2.43	3.67	-0.16	0.00	0.03	0.00	0.00	2.61	0.18	0.03
19	2.06	4.12	0.28	-0.37	0.08	0.14	-0.11	2.11	0.05	0.00
20	2.92	3.54	-0.29	0.49	0.08	0.24	-0.14	2.76	-0.16	0.02
21	2.98	3.78	-0.06	0.55	0.00	0.30	-0.03	2.50	-0.48	0.23
22	2.54	3.98	0.15	0.11	0.02	0.01	0.02	2.27	-0.27	0.08
23	2.41	4.08	0.25	-0.02	0.06	0.00	-0.01	2.15	-0.26	0.07
24	2.94	3.76	-0.08	0.51	0.01	0.26	-0.04	2.52	-0.42	0.18
25	2.12	3.77	-0.06	-0.31	0.00	0.10	0.02	2.50	0.38	0.15
26	2.12	3.94	0.11	-0.31	0.01	0.10	-0.03	2.31	0.19	0.04
27	2.55	3.70	-0.14	0.12	0.02	0.01	-0.02	2.59	0.04	0.00
28	2.12	3.89	0.06	-0.31	0.00	0.10	-0.02	2.37	0.25	0.06
29	2.31	3.63	-0.21	-0.12	0.04	0.02	0.03	2.67	0.36	0.13
30	2.29	3.66	-0.18	-0.14	0.03	0.02	0.03	2.63	0.34	0.12
31	2.21	3.80	-0.03	-0.22	0.00	0.05	0.01	2.47	0.26	0.07
32	2.68	3.62	-0.21	0.25	0.04	0.06	-0.05	2.67	-0.01	0.00
sum	77.86	122.65	0.00	0.00	1.04	3.32	-1.18	77.86	0.00	1.98
average	2.43	3.83	0.00	0.00	0.03	0.10	-0.04	2.43	0.00	0.06

Table (1) Fertility rate (y_i) and logarithm of per capita income measured in thousands of dollars (x_i) for the 32 states of Mexico in the year 2000.

3. (*40 pts*) For each of the following five scatterplots involving random samples for variables x and y:

- Determine whether x and y appear to be independent.
- Determine whether the OLS estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ are positive or negative.
- Rank the scatterplots in order of increasing R^2.

(a)



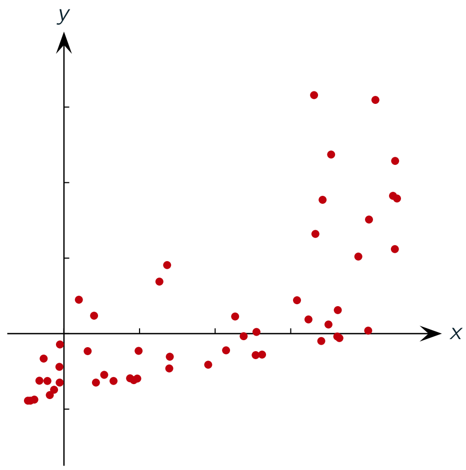
Indep: NOT independent

$\hat{\beta}_0$: Positive

$\hat{\beta}_1$: Negative

R^2: 3rd or 4th

(b)



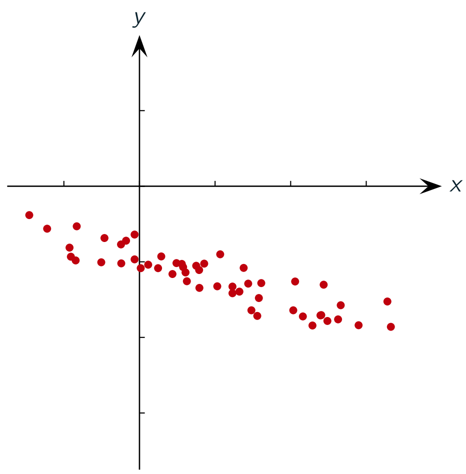
Indep: NOT independent

$\hat{\beta}_0$: Negative

$\hat{\beta}_1$: Positive

R^2: 3rd or 4th

(c)



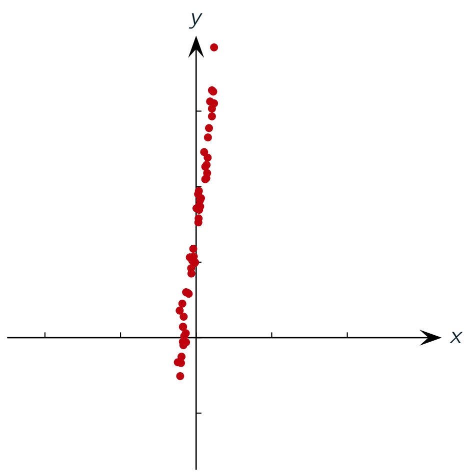
Indep: NOT independent

$\hat{\beta}_0$: Negative

$\hat{\beta}_1$: Negative

R^2: 2nd

(d)



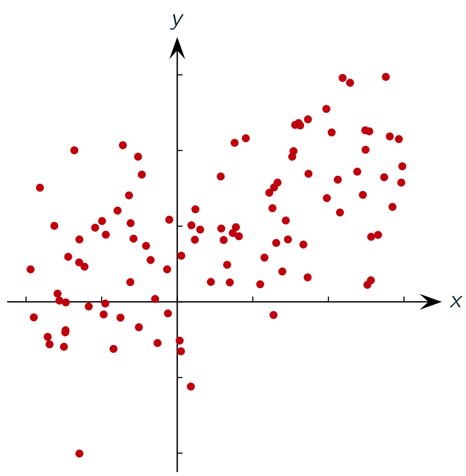
Indep: NOT independent

$\hat{\beta}_0$: Positive

$\hat{\beta}_1$: Positive

R^2: 1st

(e)



Indep: NOT independent

$\hat{\beta}_0$: Positive

$\hat{\beta}_1$: Positive

R^2: 5ft