

Solution Concepts 2

Rationalizability

Watson §7-§8, pages 67-86

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Rationalizability

- The notion of rationalizability is based on the assumption that all players are rational and agree about their rationality
- Rational players choose best responses to their beliefs about their opponent's behavior
- If they agree on their rationality, they must believe that their opponents will also choose best responses, and that their opponents believe that they will play best responses and so on and so forth
- Assuming “agreement” of rationality restricts possible beliefs to those that can be justified by a complete argument
- We say that a strategy is rationalizable if it is a best response to such beliefs

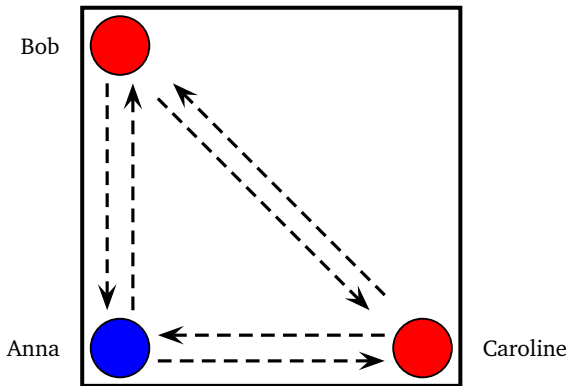
Common knowledge

Definition

We say that a fact is mutually known if everybody knows it. And we say that it is commonly known if everybody knows it, everybody knows that everybody knows it, everybody knows that everybody knows that everybody knows it, and so on and so forth

Example: three hats

Common knowledge vs mutual knowledge



[See the corresponding lecture note for further details [▶ PDF](#)]

Common knowledge of rationality

- If we assume that it is commonly known that players are rational we can make stronger predictions:
 - ① Everybody is rational
 - ⇒ players will choose best responses to *arbitrary* beliefs
 - ② Everybody knows that everybody is rational
 - ⇒ players believe that their opponents will play best responses
 - ⇒ they will choose best responses to best responses
 - ③ Everybody knows that everybody knows that everybody is rational
 - ⇒ players believe that their opponents will play best responses to best responses
 - ⇒ they will choose best responses to best responses to best responses
 - ⋮
 - ④ There is common knowledge of rationality
 - ⇒ players will choose rationalizable strategies

Iterated removal of dominated strategies

- We know that a strategy is a best response iff and only if it is not dominated \Rightarrow the previous argument is equivalent to the iterated removal of dominated strategies
- Rationality implies that we can eliminate dominated strategies (strategies that are not best responses to any belief) to obtain a smaller game
- Strategies that were not dominated in the original game can be dominated in the new game (if they are only best responses to dominated strategies)
- If everybody knows that everybody is rational we can also eliminate these strategies
- We can continue with this procedure until there are no more strategies to eliminate

Rationalizability

- The previous argument suggests the following definition of rationalizability:

Definition

A strategy is rationalizable if and only if it survives the iterated removal of strictly dominated strategies

- It can be shown that the set of rationalizable strategies is the *largest* self-rationalizable set in that: any rationalizable strategy is a best response to rationalizable strategies
- Hence a strategy is rationalizable if and only if it can be justified by a complete argument that takes into consideration the rationality of all players

Example: A 4×4 game

	a	b	c	d
w	0, 7	2, 5	7, 0	0, 1
x	5, 2	3, 3	5, 2	0, 1
y	7, 0	2, 5	0, 7	0, 1
z	0, 0	0, -2	0, 0	10, -1

- d is strictly dominated
- z is not dominated on the first stage but it is dominated once we eliminate d
- w can be rationalized by 1 using the following argument:
 - 1 believes that 2 will choose c
 - 1 believes that 2 believes that 1 will choose y
 - 1 believes that 2 believes that 1 believes that 2 will choose a
 - 1 believes that 2 believes that 1 believes that 2 believes that 1 will choose w ...

Example: location game

Description

- Henry and George are icecream vendors, they sell identical products at identical prices
- On a sunny day they must choose a location for their vending carts along the beach
- Suppose that the beach line is divided into 7 uniformly spaced regions
- On each region there are 10 customers that will buy icecream for the closest vendor (splitting evenly if the vendors are at equal distance)
- Henry and George choose their location simultaneously and their payoff is \$1 for each customer that buys from them

Example: location game

Iterated dominance

	1	2	3	4	5	6	7
1	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
2	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
3	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
4	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
5	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
6	40, 30	35, 35	30, 40	25, 45	20, 60	35, 35	60, 10
7	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

Can Henry rationalize choosing 1 or 7?

NO, because 1 is strictly dominated by 2 and 7 is strictly dominated by 6.

Example: location game

Iterated dominance

	1	2	3	4	5	6	7
1	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
2	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
3	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
4	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
5	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
6	40, 30	35, 35	30, 40	25, 45	20, 60	35, 35	60, 10
7	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

Can George rationalize choosing 2 or 6?

NO, because knowing that Henry's location will be between 2 and 6, 2 is strictly dominated by 3 and 6 is dominated by 5

Example: location game

Rationalizability

	1	2	3	4	5	6	7
1	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
2	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
3	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
4	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
5	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
6	40, 30	35, 35	30, 40	25, 45	20, 60	35, 35	60, 10
7	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

In fact the only rationalizable strategy for either player is locating at the middle of the beach, i.e. choosing 4. This result is very general and is known as *the median voter theorem*

Example: Cournot competition

Best responses

- Consider a Cournot duopoly game with two firms 1 and 2 choosing quantities $q_1, q_2 \in [0, 50]$, with constant marginal costs $c = 10$ and inverse demand function:

$$P(q_1, q_2) = 100 - q_1 - q_2$$

- Payoffs are given by:

$$u_1(q_1, q_2) = (90 - q_2 - q_1)q_1 \quad u_2(q_1, q_2) = (90 - q_1 - q_2)q_2$$

- Best responses are given by:

$$BR_1(\theta_2) = 45 - \frac{1}{2}\bar{q}_2 \quad BR_2(\theta_1) = 45 - \frac{1}{2}\bar{q}_1$$

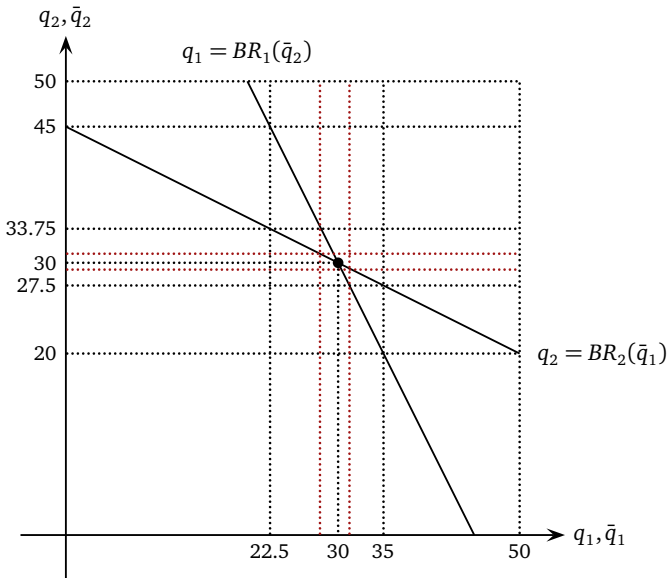
Example: Cournot competition

Rationalizability

- Firm 2's best response function only takes values between 20 and 45
- Knowing that firm 2 will choose a quantity between 20 and 45, firm 1 will only consider choosing quantities between $BR_2(20) = 35$ and $BR_2(45) = 22.5$
- Knowing that firm 1 will choose a quantity between 22.5 and 35, firm 2 will only consider choosing quantities between $BR_1(22.5) = 33.75$ and $BR_1(35) = 27.5$
- Knowing that firm 2 will choose a quantity between 27.5 and 33.75, firm 1 will only consider choosing quantities between $BR_2(27.5) = 31.25$ and $BR_2(33.75) = 28.125$
- Knowing that firm 1 will choose a quantity between 28.125 and 33.75, firm 2 will only consider choosing quantities between $BR_1(28.125) = 30.9375$ and $BR_1(31.25) = 29.375$
- If we carried out this process we will end up concluding that the only rationalizable strategy for each firm is $q_i = 30$

Example: Cournot competition

Best responses



Example: Cournot competition

Rationalizing $q_1 \neq 30$

- For firm 1 to choose $q_1 = 29$, it must believe that firm 2's average quantity is $\bar{q}_2 = 32$
- For firm 2 to choose $q_2 = 32$, it must believe that firm 1's average quantity is $\bar{q}_1 = 26$
- For firm 1 to choose $q_1 = 26$, it must believe that firm 2's average quantity is $\bar{q}_2 = 38$
- For firm 2 to choose $q_2 = 38$, it must believe that firm 1's average quantity is $\bar{q}_1 = 14$ which is never rational because firm 1 will always choose quantities above 20
- Hence firm 1 cannot rationalize choosing $q_1 = 29$

Example: Cournot competition

Rationalizing $q_1 \neq 30$

