Econ 4020 – First Preliminary Exam Practice

There are 6 questions. You have 70 minutes top answer all of them.
Justify all your answers. Good luck!

1. What is your name? Bruno Salcedo

2. What percentage grade from 0 to 100 do you think you will get on this exam? 90/100

3. Twenty three blue-eyed logicians and thirteen brown-eyed logicians are in a room. At the end of each day, those logicians who know for sure the color of their own eyes leave the room. Each logician can see the eyes of everyone else but not his own eyes. One day, a public announcement is made that "at least one logician in the room has blue eyes".

(a) How many days pass after the announcement before the first logician leaves the room? 23 days.

Justification:

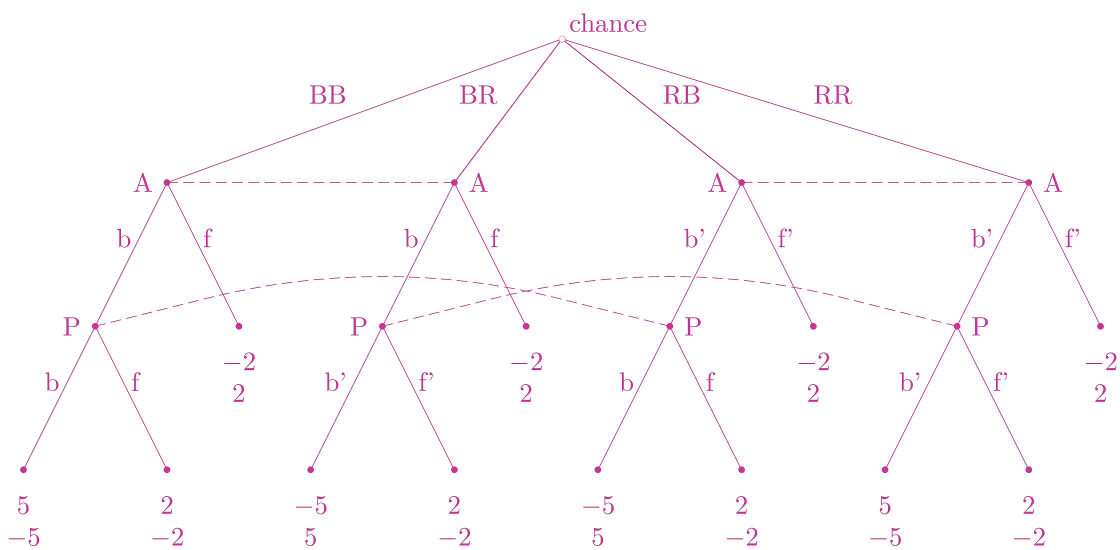
- The announcement makes it common knowledge that at least 1 logician has blue eyes
- If a person saw no logicians with blue eyes, he would know his own eyes are blue. People that don't leave the room on the **1**$^{\text{st}}$ night make it common knowledge that they *see* at least **1** logician with blue eyes
- If a person, say Bob, saw only 1 logician, say Jack with blue eyes, and Jack did not leave the room on the first day, then Bob would know that his eyes are blue (why?) and would leave on the second night. Hence, people that don't leave the room on the **2**$^{\text{nd}}$ night make it common knowledge that they *see* at least **2** logicians with blue eyes.
- Continuing this argument, people that don't leave the room on the **22**$^{\text{nd}}$ night make it common knowledge that they *see* at least **22** logicians with blue eyes.
- At this point, each logician with blue eyes can infer that he is one of the blue-eyed logicians that other blue-eyed logicians see. Hence, all blue-eyed logicians leave the room on the 23$^{\text{rd}}$ night.

(b) How many days pass after the announcement before the last logician leaves the room? 24 days.

Justification: When the blue-eyed logicians leave the room, it is because each of them could see *exactly* 22 blue-eyed logicians and knew that other blue-eyed logicians could also see 22 blue-eyed logicians. Their departure made it common knowledge that there were no more blue-eyed logicians in the room. Hence, all brown-eyed logicians leave on the next night.

4. Anna and Bob are each randomly dealt either a red or a black card facing down. Seeing the color of her own card but not that of Bob, Anna chooses whether to bet that the cards have the same color, or to fold. If Anna bets, Bob chooses whether to call Anna's bet or to fold. The first player to fold pays \$1 to his/her opponent. If no player folds and both cards have the same color, Anna wins the bet and gets \$5 from Bob. Otherwise, Anna loses the bet and has to pay \$5 to Bob.

(a) Write down an extensive form game representing this situation.



(b) How many strategies does each player have? 4 each.

(c) Write down a strategic form game representing this situation. See the table at the top of next page. The numbers correspond to expected utilities.

5. Suppose the central issue that will drive the outcome of an upcoming election is the new local tax rate. Four different rates are being considered: 1%, 1.25%, 1.5% and 1.75%. Each voter will vote for the candidate that proposes a rate closest to the one he or she prefers. There are 3600 people who want 1%, 200 that prefer 1.25%, 2400 that prefer 1.5%, and 1800 that prefer 1.75% (see the table). Suppose

	bb’	bf’	fb’	ff’
bb’	$0, 0$	$1, -1$	$1, -1$	$2, -2$
bf’	$-1, 1$	$\frac{3}{4}, -\frac{3}{4}$	$-\frac{7}{4}, \frac{7}{4}$	$0, 0$
fb’	$-1, 1$	$-\frac{7}{4}, \frac{7}{4}$	$\frac{3}{4}, -\frac{3}{4}$	$0, 0$
ff’	$-2, 2$	$-2, 2$	$-2, 2$	$-2, 2$

there are two candidates running for office, Mia and Nick, and they will propose a tax rate simultaneously and independently. The candidates do not care about tax rates, they simply want to maximize the number of votes they get.

tax rate	1%	1.25%	1.5%	1.75%
voters	3600	200	2400	1800

(a) Write down a strategic form game that represents this situation from the perspective of the candidates.

	1%	1.25%	1.5%	1.75%
1%	$4, 4$	$3.6, 4.4$	$3.7, 4.3$	$3.8, 4.2$
1.25%	$4.4, 3.6$	$4, 4$	$3.8, 4.2$	$5, 3$
1.5%	$4.3, 3.7$	$4.2, 3.8$	$4, 4$	$6.2, 1.8$
1.75%	$4.2, 3.8$	$3, 5$	$1.8, 6.2$	$4, 4$

(b) Use iterated removal of dominated strategies to determine which strategies are rationalizable.

- 1% is strictly dominated by 1.25% for both players
- 1.75% is strictly dominated by 1.5% for both players
- Once 1% is eliminated, 1.25% is strictly dominated by 1.5%
- The only rationalizable strategy for each player is 1.5%

6. Consider a strategic form game with players 1 and 2. Player 1 chooses $x \in [-2, 2]$ while player 2 chooses $y \in [0, 2]$. Payoffs are given by:

$$u_1(x, y) = -\Big(2|x| - y\Big)^2 \quad \text{and} \quad u_2(x, y) = y\left(2 + \frac{1}{2}x - y\right)$$

(a) Suppose that player 1 believes that player 2 will choose $y = 2$. What are *all* of player 1's best responses to such beliefs? $\mathrm{BR}_1(2) = \{-1, 1\}$

Justification: $u_1(1,2) = u_1(-1,2) = 0$ and $u_1(x,2) < 0$ for any $x \notin \{1,-1\}$.

(b) Graph both players best responses as a function of their beliefs in a clearly labeled figure. $\mathrm{BR}_1(\theta_2) = \{\bar{y}/2, -\bar{y}/2\}$ and $\mathrm{BR}_2(\theta_1) = 1 + \bar{x}/4$, where I am using the notation $\bar{y} = \mathbb{E}_{\theta_2}[\mathbf{y}]$ and $\bar{x} = \mathbb{E}_{\theta_1}[\mathbf{x}]$.

Justification: For player 1 it is as in part (a). For player 2, note that:

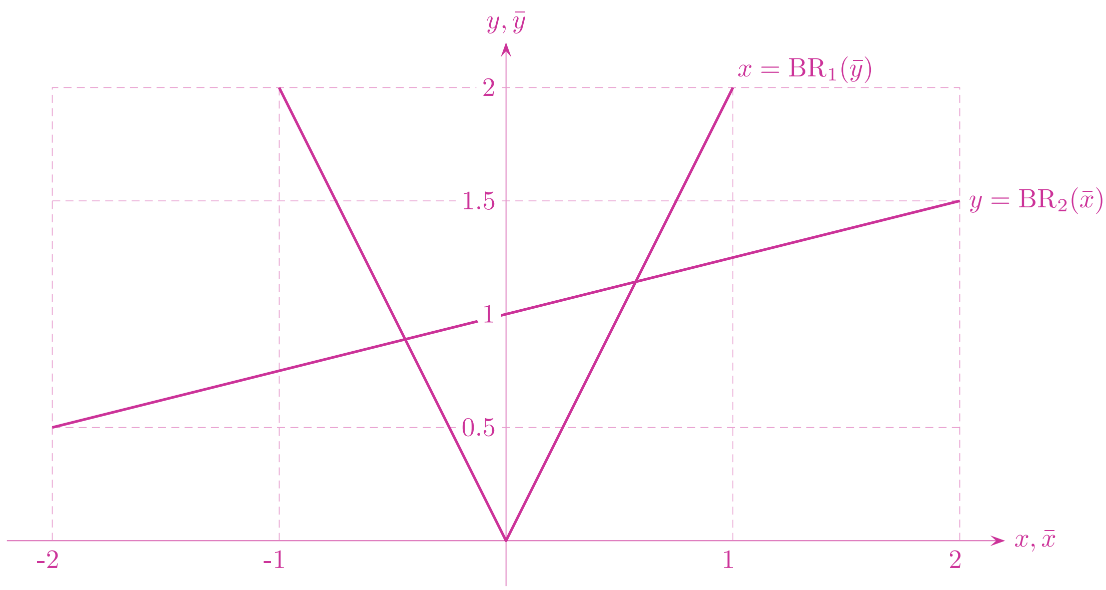
$$U_2(y, \theta_1) = -y^2 + \left(2 + \frac{1}{2}\bar{x}\right) y$$

and thus

$$U_2'(y, \theta_1) = -2y + \left(2 + \frac{1}{2}\bar{x}\right)$$

The first order condition is then $-2y1$

$$-2y + \left(2 + \frac{1}{2}\bar{x}\right) = 0 \qquad \Rightarrow \qquad y = 1 + \frac{1}{4}\bar{x}$$



(c) Find all dominated strategies for each player. For player 1 strategies $x < -1$ and strategies $x > 1$ are dominated. For player 2 strategies $y < 0.5$ and strategies $y > 1.5$ are dominated.

Justification: From the figure, BR_1 only takes values between -1 and 1, and

BR_2 only takes values between 0.5 and 0.75. From a proposition discussed in class, a strategy is strictly dominated if and only if it is never a best response.

(d) Is $y = 0$ rationalizable? Yes.

Justification:

- It is easy to verify that $4/7 \in \text{BR}_1(8/7)$ and $8/7 = \text{BR}_2(4/7)$. Hence, $x = 4/7$ and $y = 8/7$ are rationalizable
- By a similar logic, $x = -4/9$ and $y = 8/9$ are also rationalizable
- Now, suppose that player 1 believes that $x = 4/7$ with probability $\theta_2(4/7) = 7/16$, and believes that $x = -4/9$ with probability $\theta_2(-4/9) = 9/16$.
- It is easy to verify that, with such beliefs, $\mathbb{E}[\theta_2][\mathbf{x}] = 0$, and thus, $\text{BR}_2(\theta_2) = 0$.

(e) Find a rationalizable strategy for player 1. $x = 4/7$.