

Take home exam

Knowledge, belief and rationality
New Economic School and Higher School of Economics
Summer 2013

Due on 07 · 25 · 2013

Problem 1. Unawareness

“Reports that say that something hasn’t happened are always interesting to me, because as we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns – the ones we don’t know we don’t know. And if one looks throughout the history of our country and other free countries, it is the latter category that tend to be the difficult ones.”

– Donald Rumsfeld

For this problem consider the single-agent epistemic model from Dekel et al. (1998). The model consists of a finite set of states of the world Ω , and two correspondences $K, U : 2^\Omega \rightrightarrows 2^\Omega$. K is to be thought of as a knowledge correspondence, i.e. $K(E)$ is the set of states at which the agent knows the event E . U is to be thought of as an unawareness correspondence, i.e. $U(E)$ is the set of states at which the agent is unaware of the event E . Being unaware of E means that the agent does not know E , and also does not know that she does not know E . This idea is captured by the following condition:

$$\forall E \subseteq \Omega : \quad U(E) \cap K(E) = \emptyset \quad \wedge \quad U(E) \cap K(\Omega \setminus K(E)) = \emptyset \quad (U0)$$

It may also be desirable to impose the following conditions:

$$\forall E \subseteq \Omega : \quad K(U(E)) = \emptyset \quad (U1)$$

$$\forall E \subseteq \Omega : \quad U(E) \subseteq U(U(E)) \quad (U2)$$

- 1.a)** Prove that if the model satisfies (U0) and the monotonicity and truth axioms (conditions (K2) and (K3) in the class slides), then it also satisfies (U1).
- 1.b)** Prove that if the model satisfies (U0)–(U2) and monotonicity (K2), then *the agent cannot know something and be unaware of something else at the same time*, i.e. $U(E) \cap K(F) = \emptyset$ for all $E, F \in 2^\Omega$
- 1.c)** Prove that if the model satisfies (U0)–(U2) and the completeness axiom (K1), then *the agent cannot be unaware of anything*, i.e. $U(E) = \emptyset$ for all $E \subseteq \Omega$.

Problem 2. Weak dominance

For this problem consider a two player finite game in strategic form (I, A, u) , with $I = \{1, 2\}$. In class, we introduced a notion of strict dominance for such games. This problem asks you to analyze a notion of *weak dominance*, and a notion of *dominated* actions introduced by Borgers (1993). For each player i and every set $B_{-i} \subseteq A_{-i}$, define the *weak dominance* relation conditional on B_{-i} as follows. a_i weakly dominates a'_i with respect to B_{-i} if and only if:

$$\forall a_{-i} \in B_{-i} \quad u_i(a_i, a_{-i}) \geq u(a'_i, a_{-i}) \quad (\text{WD1})$$

$$\exists a_{-i} \in B_{-i} \quad u_i(a_i, a_{-i}) > u(a'_i, a_{-i}) \quad (\text{WD2})$$

We say that a_i is *weakly dominated* with respect to B_{-i} if there exists some a'_i which weakly dominates it with respect to B_{-i} . When $B_{-i} = A_{-i}$, we omit the reference to B_{-i} and simply say that a'_i weakly dominates a_i , or that a_i is weakly dominated. A strategy a_i is said to be *dominated* if and only if it is weakly dominated with respect to every $B_{-i} \subseteq A_{-i}$.

- 2.a)** Show that if a player has strictly positive beliefs (assigns strictly positive probability to his opponents playing a_{-i} for every $a_{-i} \in A_{-i}$), then his best responses cannot be weakly dominated.
- 2.b)** Provide an example of a game with a pure strategy Nash equilibrium conformed by weakly dominated strategies.
- 2.c)** Show that a dominated action can never be a best response to any belief.
- 2.d)** [Bonus] Show by example that the order of elimination of *weakly* dominated strategies matters.

References

Borgers, T. (1993). Pure strategy dominance. *Econometrica*, 61(2):423–430.

Dekel, E., Lipman, B. L., and Rustichini, A. (1998). Standard state-space models preclude unawareness. *Econometrica*, 66(1):159–173.